Board Exam Desk

Hydraulics exercises and solutions

Complete exercise and solution chapters on hydrostatics, pipe flow and open-channel flow.

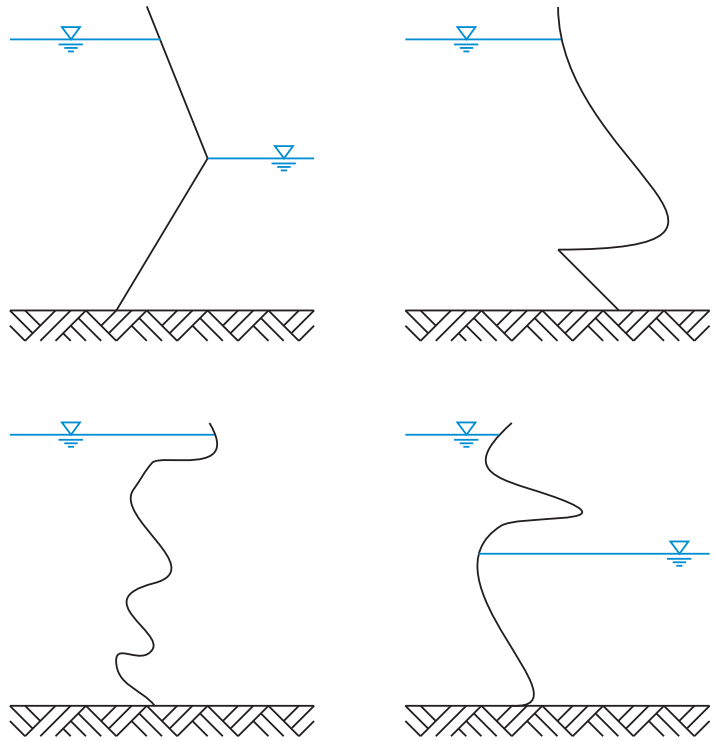
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Chapter 16
Exercises[A]

1 Hydrostatics

Exercise 1.1

Draw the horizontal and vertical pressure diagrams in the figures below.

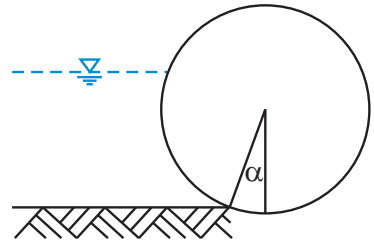


[A] The exercises were developed by the author and were set in a form similar to those in diploma examinations between 2003 and 2012 at the Technical University of Munich.

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C. Rapp, *Hydraulics in Civil Engineering*,
https://doi.org/10.1007/978-3-031-54860-4_16

Exercise 1.2

A roller dam ($D = 3.0\,\mathrm{m}$) closes at an angle of $\alpha = 20°$ with respect to vertical.

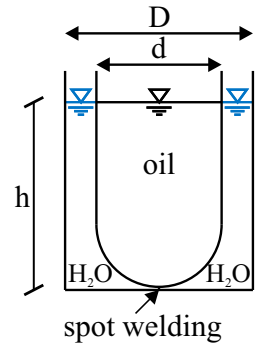


a) Draw the vertical pressure diagram for the case of maximum buoyancy.
b) Calculate the magnitude and direction of the resulting force acting on the roller.
c) Calculate the moment of force that must be counteracted by the roller axis.

Exercise 1.3

An oil-filled ($\rho_{\mathrm{oil}} = 800\,\mathrm{kg/m^3}$) rotation-symmetrical vessel with hemispherical bottom is located in a water-filled container that is also rotation-symmetrical. Answer the questions in parts a) through d) on the basis of the following values:

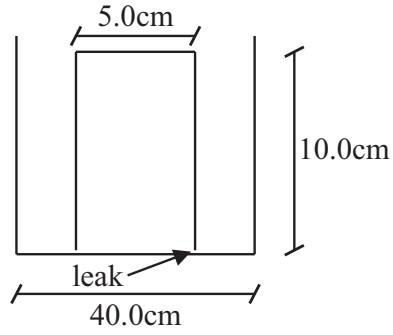
$$D = 4.0\,\mathrm{m},\ d = 2.5\,\mathrm{m},\ h = 5.0\,\mathrm{m}$$



a) Draw the resulting pressure distribution on all the walls.
b) Calculate the support force of the foundation.
c) Calculate the pressure force acting on the bottom of the water-filled container.
d) Calculate the force that the weld seam must resist.

Exercise 1.4

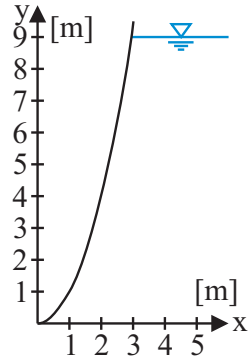
A cylindrical glass of mass 116 g and with dimensions $R = 2.5\,\mathrm{cm}$ and $h = 10.0\,\mathrm{cm}$ is placed upside-down in a square container that is filled with water. Contact between the glass and the container is not tight; hence, water may flow in and out. The atmospheric pressure amounts to $p_0 = 101\,300\,\mathrm{N/m^2}$. Assume that the glass does not tip over.



a) How much water must be poured into the container so that the glass floats?
b) Draw the pressure distribution on the glass at the moment shortly before it floats.

Exercise 1.5

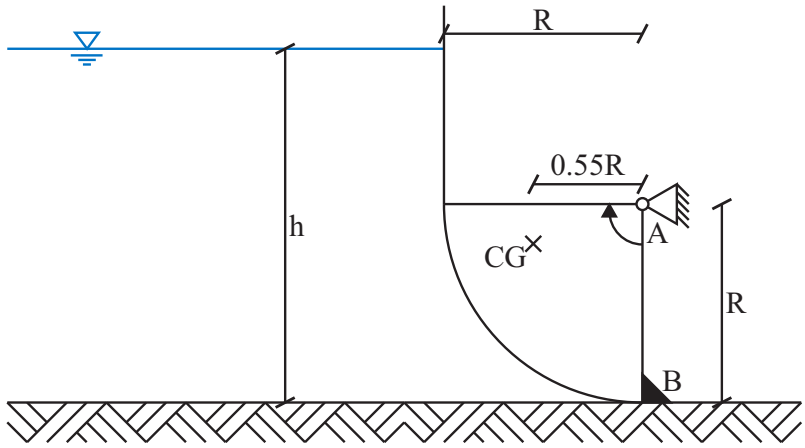
The retaining wall illustrated in the sketch below is modelled by the equation $y = x^2; x \in (0 \ldots 3)$. The water level is at 9.0 m.



a) Draw the resulting horizontal and vertical pressure diagrams on the sketch.
b) Calculate the resulting forces per unit width in the horizontal and vertical direction.
c) Calculate the magnitude and direction of the resulting force.

Exercise 1.6

The weir shown below may turn about point A and opens clockwise. The lower part of the weir has the shape of a quadrant with radius $R = 3.0\,\text{m}$. The weight force is $F_G = 30\,\text{kN/m}$ and the centre of gravity CG of the weir is shown. Forces due to friction are assumed to be negligible.



a) Draw the resulting pressure diagram at the closed weir baffle, and determine the resulting magnitude and direction of the pressure force per unit width at a water level of $h = 5.0\,\mathrm{m}$.
b) Determine the moment of force that is required to open the weir, when the water level is at $h = 5.0\,\mathrm{m}$.
c) Determine the maximum support force that the thrust bearing at point B must exert when the baffle is closed. What is the water level when this force applies?

2 Outflow through openings

Exercise 2.1

Two basins identically constructed with base area $A = 100\,\mathrm{m}^2$ are connected via a pipeline ($L = 20.0\,\mathrm{m}$; $f_\mathrm{D} = \mathrm{const.} = 0.03$; $d = 0.10\,\mathrm{m}$). Both the inlets and outlets are located at the bottom of the containers. One of the containers is situated 5.0 m higher than the other and is filled with water up to a height of $h = 2.0\,\mathrm{m}$; the lower one is completely empty. The water level of the upper basin is to be lowered to 1.0 m. How long does that take? In your calculation, neglect the acceleration of the water column as well as the water in the pipeline.

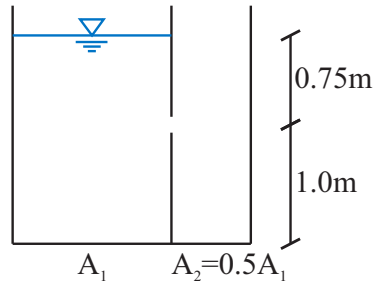
Exercise 2.2

The aeration basin of a treatment plant is to be designed for a wastewater flow rate of $0.2\,\mathrm{m}^3/\mathrm{s}$ to $1.0\,\mathrm{m}^3/\mathrm{s}$. The available base area of the basin is 20 m by 40 m. The discharge is spilled over a fixed weir ($\mu = 0.6$) along the entire short side of the basin. The minimum average dwell time of the wastewater in the basin should be assumed to be 40 min.

a) Determine the minimum required basin depth and weir height.
b) Calculate the dwell time at minimum inflow.

Exercise 2.3

A container with a constant base area $A_1 = 1.0\,\mathrm{m}^2$ is drained into an immediately adjacent container that is initially empty with cross-sectional area $A_2 = 0.5 \times A_1$. The discharge from the first container into the second is via an opening that is located $1.0\,\mathrm{m}$ above the bottom of the container (see sketch). The orifice has a cross-sectional area of $5.0\,\mathrm{cm}^2$ with a discharge coefficient of $\mu = 0.65$.



a) Where (at what height) is the balanced condition established?
b) During how much time does a free discharge take place? This may be solved without subtask a).
c) How long does it take for the balanced condition to be established?

Exercise 2.4

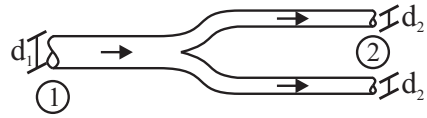
The completely drained rainwater retention basin in the Munich Hirschgarten is filled continuously during a heavy rain event at the rate of $10\,\mathrm{m}^3/\mathrm{s}$. Assume a constant base area $A = 2.0 \times 10^5\,\mathrm{m}^2$. The overflow (round crested weir) is located $10.0\,\mathrm{m}$ above the bottom of the basin ($b = 5.0\,\mathrm{m}$, $\mu = 0.70$). Accounting for a potential influence from backwater is not the object of this problem.

a) When does the rain-induced overflow start?
b) At what level of the water in the rainwater retention basin is a balanced condition established?
c) Calculate the time it takes for this condition to be reached.
d) How much untreated wastewater has entered the receiving water up to this point of time?

3 Momentum equation

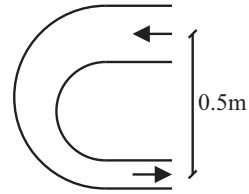
Exercise 3.1

Calculate the horizontal support force for the Y-pipe, shown in the sketch, of a hydropower plant. At cross-section 1, the pressure height is determined to be $30\,\mathrm{mH_2O}$ and the discharge is $10\,\mathrm{m}^3/\mathrm{s}$. Furthermore, the diameters $d_1 = 1.0\,\mathrm{m}$ and $d_2 = 0.60\,\mathrm{m}$ as well as the energy loss between cross-sections 1 and 2, $\Delta e_{1-2} = 2.0\,\mathrm{m}$, are given.



Exercise 3.2

Consider a U-pipe with a circular cross-section whose inlet diameter tapers from $d_I = 0.4\,\mathrm{m}$ to the outlet diameter $d_O = 0.2\,\mathrm{m}$. The pipe axes are spaced $0.5\,\mathrm{m}$ apart. For a discharge of $Q = 0.05\,\mathrm{m}^3/\mathrm{s}$, a pressure height of $0.5\,\mathrm{mH_2O}$ exists at the inlet.

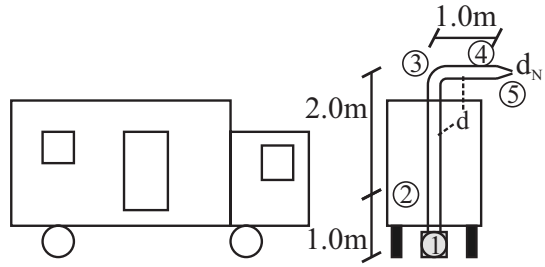


What is the magnitude of the horizontal force acting on the pipe curvature if

a) the U-pipe lies horizontally and friction losses are neglected?
b) the U-pipe stands vertically (inlet cross-section on top) and the minor (local) losses are set to $\Delta e = 0.2\left(\frac{\Delta v^2}{2g}\right)$?

Exercise 3.3

The problems below are to be solved for a water cannon used by the police. Water is withdrawn from a hydrant (location 1). Answer the following questions regarding locations 1 – 5:



$d = 0.10\,\mathrm{m}$	$d_{\mathrm{nozzle}} = 0.01\,\mathrm{m}$	$f_{\mathrm{D}} = 0.03$	$K_{\mathrm{bend}} = 0.80$	$K_{\mathrm{nozzle}} = 0.2\frac{v^2_{\mathrm{nozzle}}}{2g}$

a) Draw a completely labelled energy diagram (locations 1 – 5).
b) Determine the discharge if the pressure is $p_2 = 4.0\,\mathrm{bar}$ at location 2.

If you have not calculated the values in subtask b), you may use the following alternative values for exercises c), d) and e): discharge $Q = 4.0\,\mathrm{L/s}$; pressure height just before the nozzle at location 4 is $p_4 = 30.0\,\mathrm{m}$

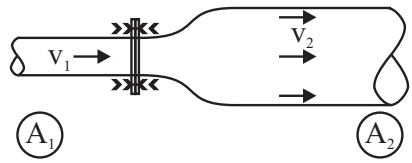
c) Calculate the resulting horizontal force acting on the police car.

d) What force is the connection between nozzle and pipe subjected to?
e) How far does the water splash? Please note the formula for the horizontal trajectory (assume a loss-free situation).

$$\vec{x}(t) = \begin{pmatrix} v_0 \cdot t \\ -\frac{1}{2} \cdot g \cdot t^2 \end{pmatrix}$$

Exercise 3.4

A horizontal pipe expansion is attached to the feed pipe by a flange. The cross-sectional area A_1, the velocity v_1 and the pressure p_1 are given.

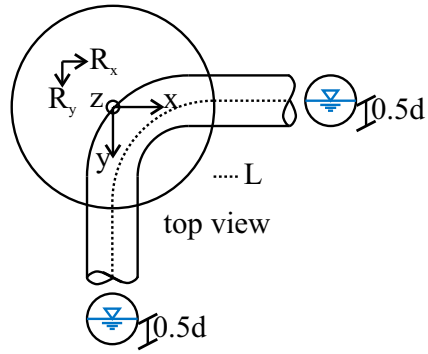


$$\boxed{A_1 = 7.00\,\text{cm}^2} \quad \boxed{v_1 = 4.00\,\text{m/s}} \quad \boxed{p_1 = 1.0 \times 10^5\,\text{N/m}^2}$$

a) First, the cross-sectional area A_2 is to be determined such that the velocity height in A_2 corresponds to one-seventh of the velocity height in cross-sectional area A_1.
b) Now calculate the pressure in cross-sectional area A_2. Take the energy losses into account: $K_{\text{exp}} = 0.5$ (related to v_2).
c) What is the magnitude of the horizontal force acting on the flange?

Exercise 3.5

Water in a channel with a circular cross-section is redirected with a deflection angle of 90° in a shaft structure. At a discharge of 40 L/s, the flow depth upstream from the shaft corresponds to the flow depth downstream. The pipe diameter is 30.0 cm and the flow depth was measured as 15.0 cm.

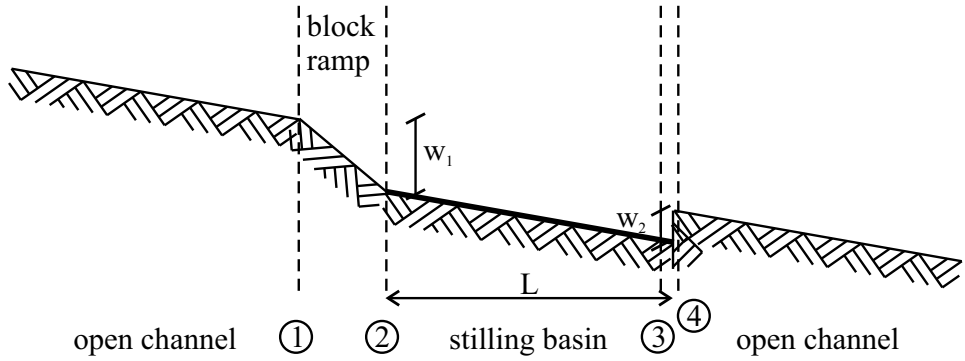


a) What type of flow exists (sub- or supercritical flow)?
b) What is the resulting horizontal force counteracted by the shaft?

4 Free surface flows

Exercise 4.1

A sketch of a rectangular open channel section with a width of 5.0 m and a discharge of $20\,\mathrm{m^3/s}$ is shown below. A stilling basin with a concrete bottom is arranged downstream from a block ramp with $w_1 = 1.5\,\mathrm{m}$. The stilling basin ends with a vertical step $w_2 = 0.50\,\mathrm{m}$.



$Q = 20\,\mathrm{m^3/s}$	$w_1 = 1.5\,\mathrm{m}$	$n_{\mathrm{sb}} = 1.43 \times 10^{-2}\,\mathrm{s/m^{\frac{1}{3}}}$	$J_{B,\mathrm{sb}} = 0.8\,\%$	$\Delta e_{1-2} = \frac{v_c^2}{2g}$
$b = 5.0\,\mathrm{m}$	$w_2 = 0.5\,\mathrm{m}$	$n_{\mathrm{channel}} = 2.86 \times 10^{-2}\,\mathrm{s/m^{\frac{1}{3}}}$	$J_{B,\mathrm{channel}} = 0.8\,\%$	$\Delta e_{3-4} = \frac{0.1 \cdot v_{N,\mathrm{channel}}^2}{2g}$

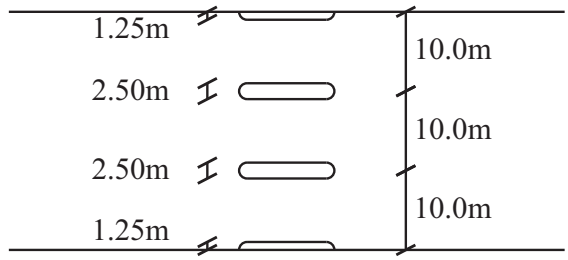
a) Calculate the normal water depths, the critical depth and the corresponding specific energies in all areas.
b) Calculate the length L in such a manner that the hydraulic jump does not quite reach into the area of the block ramp.

$$L_{\mathrm{HJ}} = 8.5 y_l \left(\mathrm{Fr}_l - 1.7\right)$$

c) Draw a completely labelled energy diagram.

Exercise 4.2

The bridge over the Isar River in Munich-Thalkirchen is to be examined for a possible flood situation. In this area, the Isar may be approximated by a rectangular cross-section (width $b = 30.0\,\mathrm{m}$). Make the assumptions regarding measurements and quantities as indicated in the following sketch:

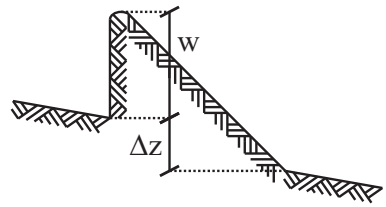


$Q = 500\,\mathrm{m^3/s}$	$b = 30.0\,\mathrm{m}$	$J_B = 0.3\,\%$	$n = 2.86 \times 10^{-2}\,\mathrm{s/m^{\frac{1}{3}}}$

a) Calculate the normal water conditions.
b) Due to a log jam in the confined area, a loss of $K = 0.5$ related to the critical velocity in the constriction, $v_{c,\mathrm{cons}}$, occurs. Determine the course of the water level and draw the energy diagram.
c) Calculate the total length of the influence of the log-jammed bridge on the flow.

Exercise 4.3

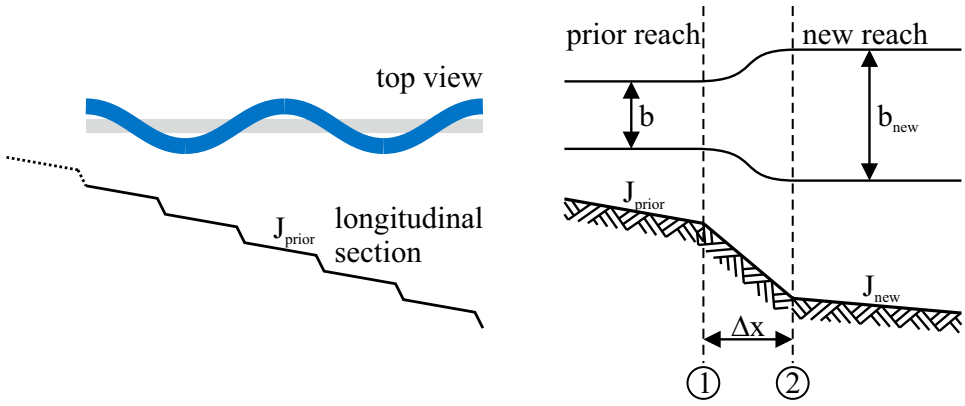
A weir ($\mu = 0.60$, $w = 1.50\,\mathrm{m}$, $\Delta z = 1.0\,\mathrm{m}$) is built in a channel with rectangular cross-section ($b = 8.0\,\mathrm{m}$) and a discharge of $Q = 40\,\mathrm{m^3/s}$. At the tail of the weir, the open channel width changes to $b = 12.0\,\mathrm{m}$. The backwater influence need not be examined. The downstream normal water flow depth amounts to $y_N = 1.35\,\mathrm{m}$.



a) Calculate the overflow height as well as the flow depths both upstream and downstream from the weir.
b) A hiker measures the distance between the tail of the weir and the toe of the hydraulic jump as 20.0 m. Calculate the Manning coefficient and the gradient of the bed for this open channel section.

Exercise 4.4

An existing open channel with rectangular cross-section ($b = 5.0\,\mathrm{m}$) is to be developed over a length of 5.0 km in a more natural way (renaturated). To stabilise the bed, the original open channel has a bottom step of 1.0 m every 1000 m along its length. Assume that energy losses of $0.1 \cdot k_c$ occur in the area of the bottom step. The new open channel is to be expanded to a width of $b_{\mathrm{new}} = 8.0\,\mathrm{m}$ and provided with twice the flow length by the meanders. Secondary flows are not to be taken into account. The roughness of the bed will not change.



$Q = 15\,\mathrm{m^3/s}$	$b = 5\,\mathrm{m}$	$n = 2.86 \times 10^{-2}\,\mathrm{s/m^{\frac{1}{3}}}$	$J_{B,\mathrm{prior}} = 0.5\,\%$	$b_{\mathrm{new}} = 8\,\mathrm{m}$

a) Evaluate whether bottom steps are required for the new open channel. Note: the mean flow velocity must not increase; normal water conditions are to be considered.

In the following subtasks, only the transition from the existing to the renaturated open channel is to be considered (see right-hand figure).

b) Quantitatively specify the course of the water level along the illustrated flow sections (right-hand figure). There is no air entrainment at the bottom step.

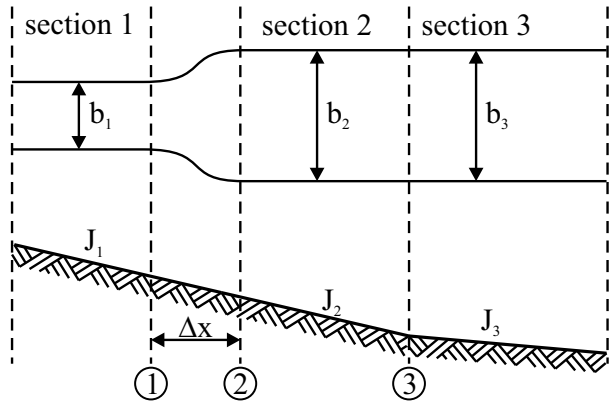
c) Specify the length of the required bottom reinforcement in the renaturated section. The length of the hydraulic jump may be calculated by means of the following formula:

$$L_{\mathrm{HJ}} = 8.5 y_l \left(\mathrm{Fr}_l - 1.7\right)$$

d) Draw an energy diagram of the region where the existing geometry connects to the renaturated one (see right-hand figure).

Exercise 4.5

A channel with rectangular cross-section widens from b_1 to b_2. The first part of the open channel has gradient $J_{B1} = J_{B2}$ until point 3. From there, the open channel has slope J_{B3}. In each of the sections 1 – 3, normal water conditions are attained.

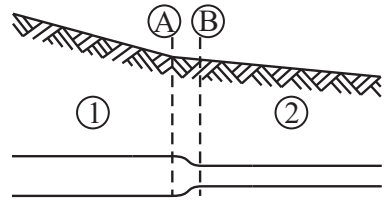


$Q = 2.0\,\mathrm{m}^3/\mathrm{s}$	$b_1 = 1.0\,\mathrm{m}$	$n_1 = 3.33 \times 10^{-2}\,\mathrm{s/m}^{\frac{1}{3}}$	$J_{B1} = 2.0\,\%$
$\Delta e_{1-2} = \Delta x \cdot J_{B1}$	$b_2 = 4.0\,\mathrm{m}$	$n_2 = 2.5 \times 10^{-2}\,\mathrm{s/m}^{\frac{1}{3}}$	$J_{B2} = 2.0\,\%$
	$b_3 = 4.0\,\mathrm{m}$	$n_3 = 2.5 \times 10^{-2}\,\mathrm{s/m}^{\frac{1}{3}}$	$J_{B3} = 1.0$

a) Calculate the normal conditions for all sections.
b) Specify the flow conditions in the individual sections. Substantiate the flow depths in cross-sections 1 – 3.
c) Draw a complete energy diagram that contains all characteristic specific energies and flow depths.

Exercise 4.6

A rectangular channel ($Q = 40.0\,\mathrm{m}^3/\mathrm{s}$) is depicted below. It tapers between cross-sections A and B (distance 5.0 m) from width b_1 to b_2. The tapering causes an energy loss of $\Delta e = 0.1 \cdot k_{N2}$ (k_{N2} = velocity height of the normal conditions in section 2). Simultaneously, slope and roughness change at cross-section A. In both sections, normal conditions occur. Complete parts a), b) and c) using the given quantities.



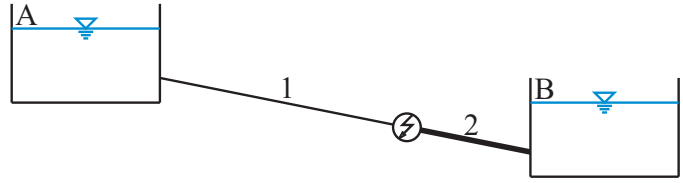
section 1	$J_{B1} = 1.5\,\%$	$n_1 = 0.025\,\mathrm{s/m}^{\frac{1}{3}}$	$b_1 = 12.0\,\mathrm{m}$
section 2	$J_{B2} = 0.3\,\%$	$n_2 = 0.020\,\mathrm{s/m}^{\frac{1}{3}}$	$b_2 = 8.0\,\mathrm{m}$

a) Calculate the normal depths with their associated specific energies and flow types in the open channel sections.
b) Determine the course of the water level.
c) Draw a complete energy diagram.

5 Pipe flow

Exercise 5.1

The sketch shows a hydropower plant. Upstream and downstream from the turbine, pipes with different diameters are installed.

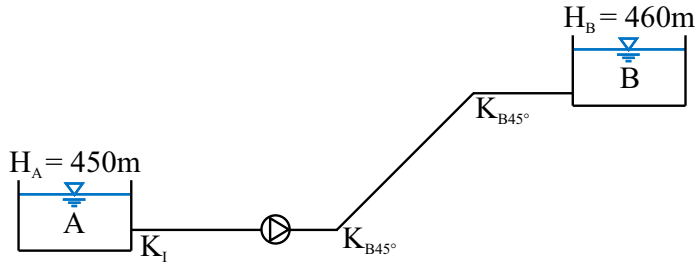


$L_1 = 2000\,\text{m}$	$d_1 = 1.0\,\text{m}$	$f_{D1} = 0.025$
$L_2 = 1000\,\text{m}$	$d_2 = 1.5\,\text{m}$	$f_{D2} = 0.020$
$H_A = 1050\,\text{m}$	$H_B = 300\,\text{m}$	$\eta_T = 0.80$

a) For what discharge does the power of the turbine P_T reach the maximum value?
b) What net head is available?
c) What power does the turbine reach?

Exercise 5.2

A pipeline is to be laid between two containers. For the transport of water from container A to container B, a pump that is to operate an average of 10 hours per day must be installed. Assume the price of electricity is 0.15 €/kWh.



$L = 5000\,\text{m}$	$K_I = 0.5$	$K_{B45^\circ} = 0.3$	$k_s = 3.0\,\text{mm}$	$T = 10\,^\circ\text{C}$	$\eta_P = 0.80$

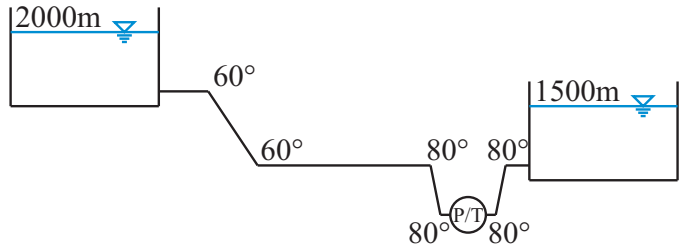
a) Determine the pump costs per year for a pipe with diameter $d = 1.5\,\text{m}$ and a discharge of $0.7\,\text{m}^3/\text{s}$.

The construction costs for one metre of pipeline, depending on the diameter d, amount to $d \cdot 7000\,€/\text{m}$.

b) Determine the optimum (most cost-effective) diameter of the pipe for $Q = 0.7\,\text{m}^3/\text{s}$ assuming that the construction is designed to last 30 years. The friction coefficient f_D from subtask a) may be used in subtask b) (substitute result $f_D = 0.024$).

Exercise 5.3

A pumped storage power plant is shown below.

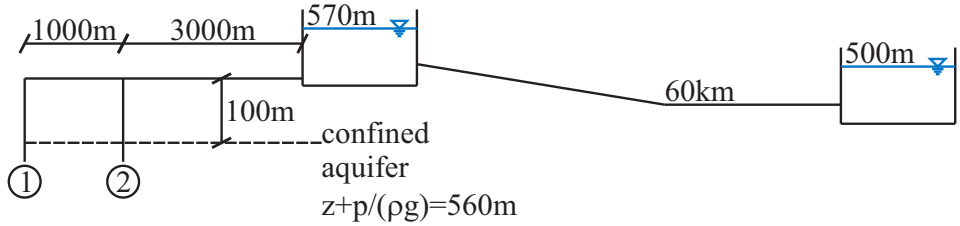


$d = 2.0\,\text{m}$	$L = 5000\,\text{m}$	$\eta_{T/P} = 0.9$	$K_{B60^\circ} = 0.5$	$K_{B80^\circ} = 0.7$	$k_s = 0.1\,\text{mm}$	$T = 5\,^\circ\text{C}$

a) Calculate the design discharge if a power of 92.9 MW is to be tapped at the turbine. Note: Start the f_D iteration with $f_{D,\,\text{step }0} = 0.01$.
b) Draw a labelled energy diagram.
c) Calculate the discharge for a pump that consumes 50 MW of electrical power using the friction coefficient of the pipe, f_D, that was determined in subtask a).
d) What is the maximum geodetic height at which the pump turbine can be installed while avoiding cavitation? Assume an energy height of 1500 m.a.s.l. and local velocities in the pump turbine of 25 m/s.

Exercise 5.4

A simplified sketch of the drinking water supply system for the city of Munich near Oberau is shown below. Water is pumped from the confined aquifer into a large tank. From there, the water, driven only by the pressure gradient, flows to the reservoir in Forstenrieder Park. Perform the calculations requested in subtasks a) and b).

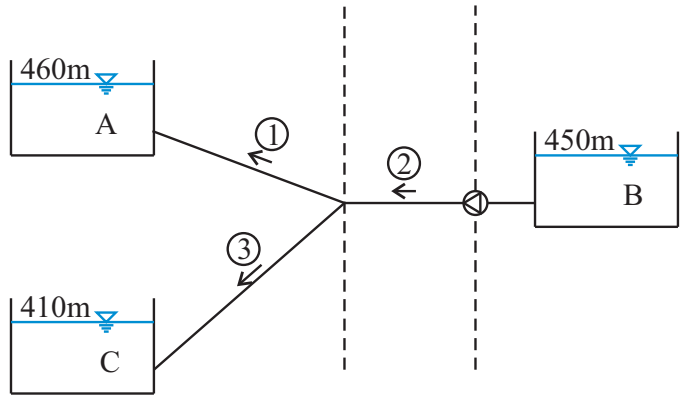


$T = 5.0\,°\text{C}$	$d_{\text{all}} = 1.40\,\text{m}$	$k_s = 0.5\,\text{mm}$	$\eta_P = 0.7$	$K_I = 0.5$

a) Calculate the steady discharge in the pipeline to Munich. Use the pipe friction coefficient determined herein for subtask b).
b) Two identical pumps that consume the same amount of power are to be employed for the two feed pipes. Calculate the partial discharges in feed pipes ① and ② as well as the power consumption of the pumps for the discharge that was determined in the previous exercise.

Exercise 5.5

A pipeline system with three containers is shown below. Water is pumped from container 2 into the two containers 1 and 3. You may refer to further data from the table below.



$H_1 = 460\,\text{m}$	$P_P = 500\,\text{kW}$	$l_1 = 3000\,\text{m}$	$d_1 = 1.00\,\text{m}$	$f_{D1} = 0.018$
$H_2 = 450\,\text{m}$	$\eta_P = 0.85$	$l_2 = 2000\,\text{m}$	$d_2 = 1.00\,\text{m}$	$f_{D2} = 0.020$
$H_3 = 410\,\text{m}$	$K_I = 0.5$	$l_3 = 2000\,\text{m}$	$d_3 = 0.30\,\text{m}$	$f_{D3} = 0.020$

a) Draw a complete energy diagram.
b) Specify the equations for the discharges in lines 1, 2 and 3 if the pump power consumption is $P_P = 500\,\text{kW}$. Specify the equation for the pressure height at the junction.

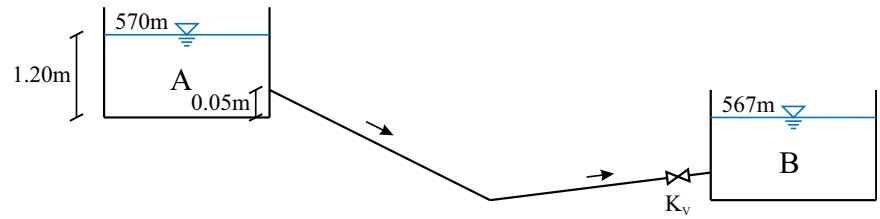
$10\,000\,\text{m}^3$ water per day are to be delivered into container 3.

c) How much water flows into container 1 during the operating time of the pump?
d) What are the electricity costs per day? Assume $0.20\,€$ for $1.0\,\text{kWh}$.

6 Cross-cutting issues

Exercise 6.1

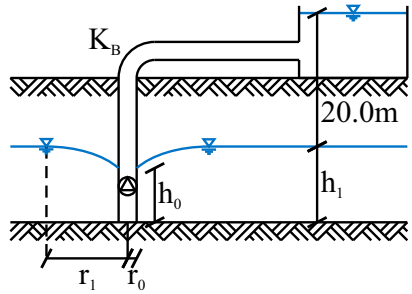
The water supply for a village in the Ecuadorian rain forest is to be investigated. Consider the idealised system illustrated in the following figure. The upper tank that is initially filled up to 570 m.a.s.l. is continuously supplied with water at a rate of 0.9 L/s from a mountain spring. The discharge from tank A with base area $A_A = 1.35\,\text{m}^2$ is controlled by the consumers in the village (pipeline data: $d = 4.0\,\text{cm}$, $l = 100\,\text{m}$, $f_D = 0.02$). The water level in tank B remains constant.



a) What loss must the valve exhibit in order to keep the water level in tank A constant?
b) Calculate the time that elapses until the water level in tank A has dropped to the inflow of the pipe when the valve is fully open ($K_V = 0$).

Exercise 6.2

For a drinking water supply, unconfined groundwater is to be accessed. Cold water at 5 °C is delivered via a pipeline of length 8.0 km into a tank.



$\eta_P = 0.80$	$K_B = 0.4$	$d_{\text{pipe}} = 0.40\,\text{m}$	$k_s = 1.0\,\text{mm}$
$h_1 = 40.0\,\text{m}$	$r_0 = 0.20\,\text{m}$	$r_1 = 3000\,\text{m}$	$k_f = 1.0 \times 10^{-4}\,\text{m/s}$

a) How much water could initially be delivered (without lowering the groundwater level (that is, $h_0 = h_1$)) if the pump consumes 7.5 kW of electrical power?

Use the friction coefficient determined in question a) for the following subtasks of the problem. Bear in mind that the groundwater level decreases due to withdrawal (see following equation).

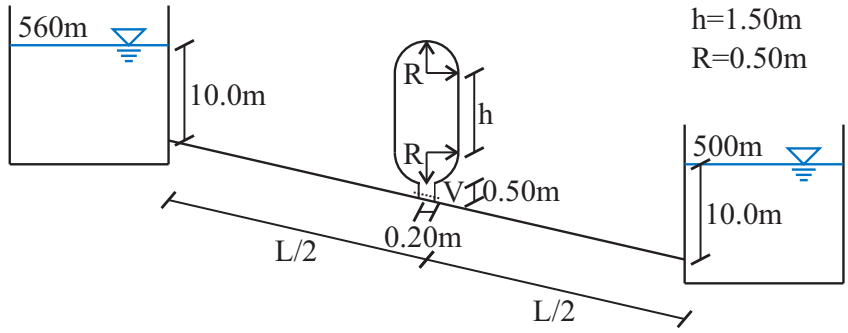
$$Q = \pi \cdot k_f \frac{h_1^2 - h_0^2}{\ln\left(\frac{r_1}{r_0}\right)}$$

b) Replace the discharge in the energy equation with the above expression.
c) How much water actually flows into the tank?
d) How far does the groundwater level drop?

Exercise 6.3

The pipeline shown below has a constant slope. The valve V of the rotation-symmetrical air vessel ($p_A = 1013\,\text{mbar}$) is initially closed. The following subtasks of the problem are to be solved for the respective steady-state cases, and $T = \text{const}$.

a) Calculate the pressure in the pipe where the air vessel is located.
b) How much would the water level in the air vessel increase if the valve were opened?
c) How does the water level change qualitatively when throttling the discharge at the lower tank?



$L = 8000\,\text{m}$	$d_{\text{pipe}} = 0.50\,\text{m}$	$f_{\text{D}} = 0.020$

Exercise 6.4

A trash rack of width b is positioned at the rectangular headwater of a hydropower plant. In the upstream water, normal conditions prevail. The minor energy losses K were measured at the rack depending on v_{upstream}. Assume that the discharge per unit width does not change in the vicinity of the rack.

a) What water depth results downstream from the rack?
b) What is the magnitude of the horizontal forces acting on the rack?

$J_B = 2.0\,\%$	$b = 7.0\,\text{m}$	$Q = 30\,\text{m}^3/\text{s}$	$K = 0.2$	$n = 0.02\,\text{s}/\text{m}^{\frac{1}{3}}$

7 Unsteady free surface flows

Exercise 7.1

In a rectangular channel of width $b = 7.50\,\text{m}$, each of two gauges that are spaced $2.0\,\text{km}$ apart indicate a flow depth of $y = 2.10\,\text{m}$. Separated by a time interval of $589\,\text{s}$, both jump to $y = 2.55\,\text{m}$. Specify:

a) the propagation velocity of the disturbance;
b) the original discharge;
c) the change in the discharge;
d) the name of the phenomenon.

Exercise 7.2

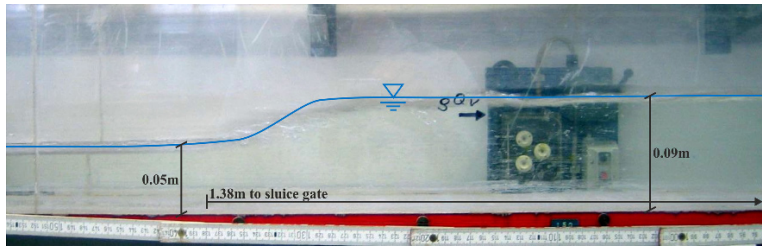
At the Finsing power plant located at the Mittlere Isar Kanal, the discharge must be reduced abruptly from $55\,\text{m}^3/\text{s}$ to $30\,\text{m}^3/\text{s}$ (assume that we are dealing with an idealised rectangular cross-section). Solve the following subtasks taking into account the given conditions:

$$\boxed{Q_0 = 55\,\mathrm{m^3/s}} \boxed{\Delta Q = -25\,\mathrm{m^3/s}} \boxed{y_0 = 6.0\,\mathrm{m}} \boxed{b = 9.0\,\mathrm{m}}$$

a) Calculate the propagation velocity of the disturbance.
b) State whether the freeboard of 1.0 m is maintained during the regulation process if it was 1.5 m in the steady-state condition.
c) When does the regulation process have an influence on the water level at the weir in Oberföhring situated 8.0 km upstream?

Exercise 7.3

The following photo was taken after the sudden closure of the sluice gate in a laboratory flume (upstream from the sluice gate). The width of the open channel is 0.20 m, and the steady-state discharge was 5.0 L/s.



a) What was the steady-state flow velocity?
b) Record the height of the surge seen in the photo.
c) Calculate the propagation velocity of the surge wave.
d) What is the magnitude of the propagation velocity of the surge wave?
e) When was the photo taken?
f) What is the percentage of the discharge reduction?

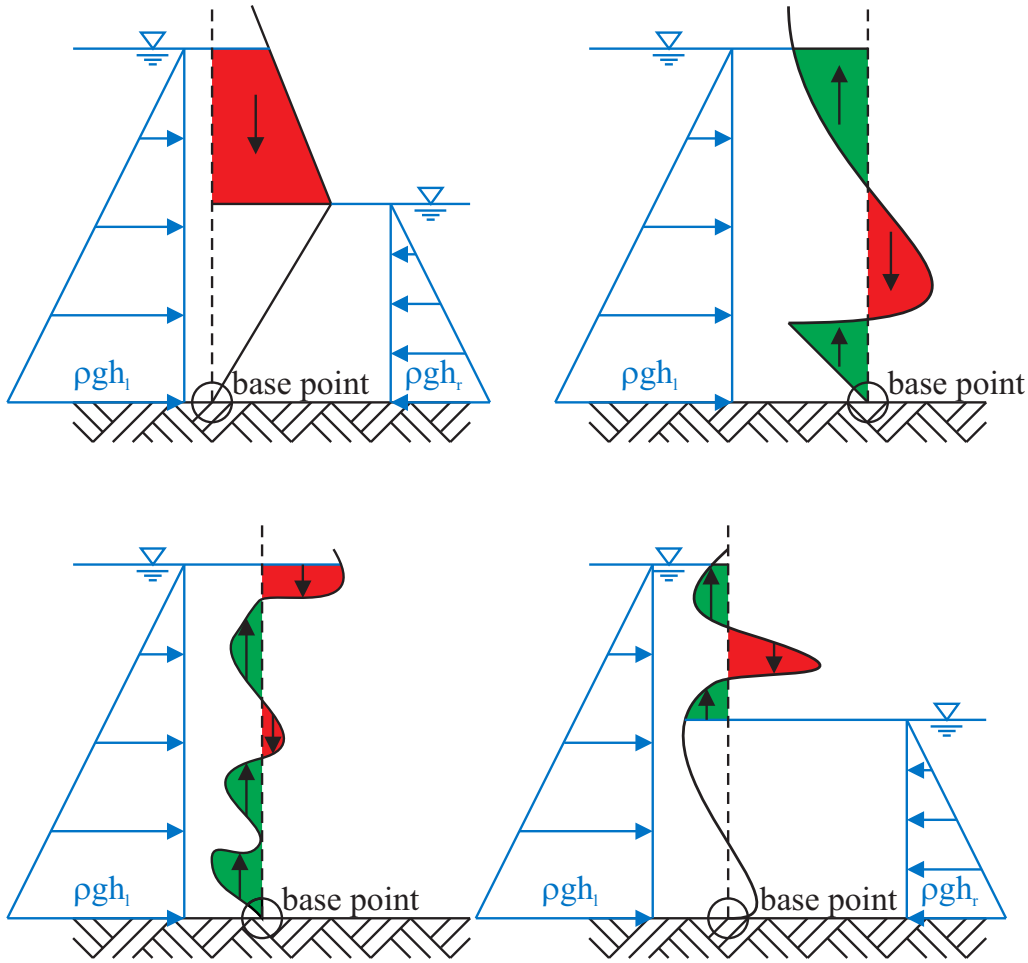
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Chapter 17
Solutions

1 Solutions: hydrostatics

Solution 1.1

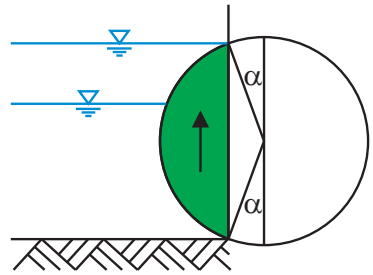
Draw a vertical line from the base point as illustrated below:



© The Author(s) 2024
C. Rapp, *Hydraulics in Civil Engineering*,
https://doi.org/10.1007/978-3-031-54860-4_17

Solution 1.2

a) Preferably, draw a vertical line from the base point.



As such, it will become obvious where the maximum buoyancy results. The water level may rise up to the height where the vertical line from the base point intersects the roller dam. This is, of course, the case at an angle of 20° relative to vertical.

b) For the horizontal force, we first need the height:

$$\cos 20^\circ = \frac{h}{2 \cdot 1.5} \rightarrow h = 2.819\,\mathrm{m}$$

$$F_{H\rightarrow} = \frac{1}{2}\rho g h^2 = \frac{1}{2}\rho g \cdot 2.819^2 = 3.898 \times 10^4\,\mathrm{N/m}$$

The buoyancy area of the circle segment may be calculated via the area of the sector minus the area of the triangle; this corresponds to the green area in the diagram.

$$A_{\mathrm{segment}} = A_{\mathrm{sector}} - A_{\mathrm{triangle}} = R^2\pi\frac{140^\circ}{360^\circ} - \frac{1}{2}hR\sin 20^\circ = 2.026\,\mathrm{m}^2$$

With this, the buoyancy force is calculated:

$$F_{V\uparrow} = \rho g A_{\mathrm{segment}} = \rho g \cdot 2.026\,\mathrm{m}^2 = 1.987 \times 10^4\,\mathrm{N/m}$$

The resultant force that is applied at $\operatorname{atan}\frac{F_V}{F_H} = 27.01^\circ$ with respect to the horizontal direction has a magnitude of $|F| = \sqrt{F_V^2 + F_H^2} = 4.375 \times 10^4\,\mathrm{N/m}$.

c) The moment caused by the water pressure may be calculated via the centres of gravity of the areas that represent the vertical and the horizontal pressure force. Thereby, the lever arm of the horizontal force is spaced two thirds of the way from the water surface to the bottom, i. e. $\frac{1}{6}h = 0.470\,\mathrm{m}$ from the centre of the circle towards the bottom. The lever arm of the buoyancy area corresponds to the distance between its centre of gravity and the centre of the circle. This is (for example, as seen in [52, p. 2.12]):

$$x_{CG,\mathrm{segment}} = \frac{h^3}{12A} = 0.922\,\mathrm{m}$$

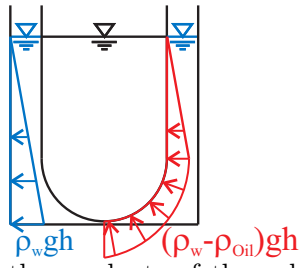
Thus, the clockwise-rotating moment of force about the centre of the circle becomes:

$$M_{\circlearrowright} = F_V \cdot x_{CG,\mathrm{segment}} - F_H\frac{1}{6}h = 0.0\,\mathrm{N\,m/m}$$

We can do without all this calculation if we just think a little. The sum of the moments about the centre is – of course – zero because the pressure is applied perpendicularly along the boundary and is therefore directed towards the centre at each point of the roller surface.

Solution 1.3

a)



b) The weight is determined by the products of the volumes of the oil and the water with their respective densities:

$$V_{\text{oil}} = \frac{2}{3}\left(\frac{d}{2}\right)^3 \pi + \left(h - \frac{d}{2}\right)\left(\frac{d}{2}\right)^2 \pi = 22.498\,\text{m}^3$$

$$V_{\text{water}} = \left(\frac{D}{2}\right)^2 \pi \cdot h - V_{\text{oil}} = 62.832 - 22.498 = 40.333\,\text{m}^3$$

$$F_{\text{foundation}} = \rho_{\text{water}} \cdot g \cdot V_{\text{water}} + \rho_{\text{oil}} \cdot g \cdot V_{\text{oil}} = 5.722 \times 10^5\,\text{N}$$

c) The force due to pressure acting on the bottom corresponds to the pressure applied there times the area.

$$F_P = \rho_{\text{water}} \cdot g \cdot h \cdot \left(\frac{D}{2}\right)^2 \pi = 6.164 \times 10^5\,\text{N}$$

d) The weld seam must absorb the resulting force because otherwise the oil container would float.

$$F_{\text{weld seam}} = F_P - F_{\text{foundation}} = 4.415 \times 10^4\,\text{N}$$

Solution 1.4

We write down the air pressure in the glass:

$$p_1 = p_0 + \rho g \Delta h$$

The gravity force of the glass, that must be counteracted, is:

$$F_G = m \cdot g = 0.116\,\text{kg} \cdot 9.81\,\text{m/s}^2 = 1.138\,\text{N}$$

The downward-directed force due to atmospheric pressure is written as:

$$F_{p0} = p_0 A = p_0 R^2 \pi$$

The upward-directed force due to pressure within the glass is generally expressed as:

$$F_{p1} = p_1 R^2 \pi$$

a) The balance of forces for our problem reads as follows:

$$p_0 A + mg = (p_0 + \rho g \Delta h) A$$

$$\Delta h = \frac{mg}{\rho g A} = \frac{0.116}{1000 \cdot 0.025^2 \pi} = 0.0591\,\text{m}$$

This yields the pressure within the glass:

$$p_1 = 101\,300 + \rho \cdot g \cdot 0.0591 = 101\,880\,\text{N/m}^2$$

And with pressure p_1, the volume $V_1 = \frac{p_0 V_0}{p_1}$ may of course be calculated and thereby also the height of the air-filled space h_1:

$$h_1 = \frac{V_1}{A} = \frac{101\,300 \cdot 0.10 \cdot R^2 \pi}{101\,880 \cdot R^2 \pi} = 0.0994\,\text{m}$$

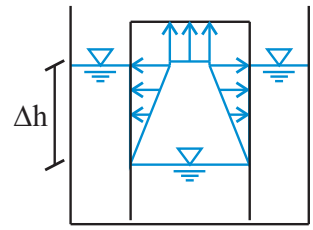
Thus, the water volume in the glass is:

$$V_{\text{water in the glass}} = 0.025^2 \pi\,(0.10 - 0.0994) = 1.12 \times 10^{-6}\,\text{m}^3$$

In total, the following volume must be filled; the entire area is wetted to $0.10\,\text{m} - 0.0994\,\text{m}$, additionally, the area outside of the glass is filled farther by Δh:

$$V_{\text{water total}} = 0.40^2\,(0.10 - 0.0994) + \left(0.40^2 - 0.025^2 \pi\right) \Delta h = 9.43 \times 10^{-3}\,\text{m}^3$$

b)



Solution 1.5

a) I'm confident that you can find the solution. Just focus on finding the **base** point.
b) You can certainly shake the horizontal pressure force out of your sleeve:

$$F_{H\leftarrow} = \frac{1}{2} \rho \cdot g \cdot 9^2 = 3.973 \times 10^5\,\text{N/m}$$

The vertical pressure force per unit width is determined via the buoyancy area:

$$F_{V\uparrow} = \rho \cdot g \cdot \frac{2}{3} \cdot 3 \cdot 9 = 1.766 \times 10^5\,\text{N/m}$$

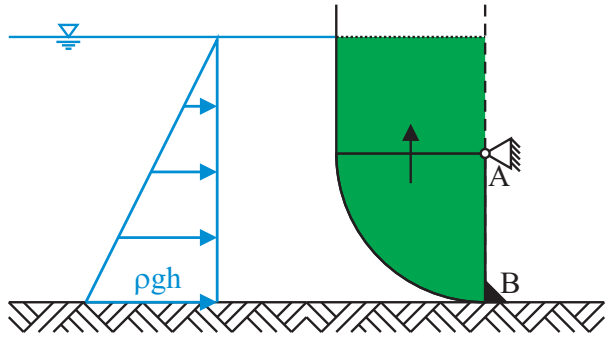
c) The magnitude of the force and its direction are obtained via the components of the force vector:

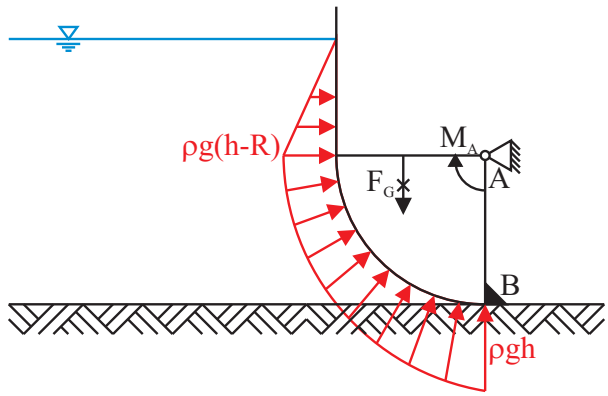
$$|F| = \sqrt{F_V^2 + F_H^2} = 4.348 \times 10^5 \,\mathrm{N/m}$$

$$\alpha = \operatorname{atan} \frac{F_V}{F_H} = 23.96^\circ$$

Solution 1.6

a)





$$F_H = \frac{1}{2}\rho g h^2 = 1.226 \times 10^5 \,\mathrm{N/m}$$

$$F_V = \rho g \left(\frac{1}{4} R^2 \pi + R\,(h - R) \right) = 1.282 \times 10^5 \,\mathrm{N/m}$$

$$|F| = 1.774 \times 10^5 \,\mathrm{N/m}$$

$$\beta = \operatorname{atan} \frac{F_V}{F_H} = 46.27^\circ$$

b) The water pressure acting upon the quadrant is radial and therefore has no moment about point A; only the portion that acts on the vertical wall causes a moment.

$$\begin{aligned} M_{\circlearrowleft} &= \left(\frac{1}{2}\rho g\,(h-R)^2\right)\frac{1}{3}\,(h-R) - F_G \cdot 0.55 \cdot R \\ &= \left(\frac{1}{2}\rho g\,(5-3)^2\right)\frac{1}{3}\,(5-3) - 3.0\times 10^4 \cdot 0.55 \cdot 3.0 = -3.642\times 10^4\,\mathrm{N\,m/m} \end{aligned} \tag{17.1}$$

c) The force that must be counteracted at point B is constant for all $0 < h < R$ because the lines of action of the pressure acting on the quadrant pass through A. If the water level rises beyond the quadrant, the portion above it turns clockwise around A, which reduces the bearing force F_B. Thus, the stabilisation of the dead weight must be compensated at point B:

$$F_{B,\max} = \frac{F_G \cdot 0.55 \cdot R}{R} = 16.5\times 10^3\,\mathrm{N/m}$$

2 Solutions: outflow through openings

Solution 2.1

Well, this looks a bit difficult, although we should be happy that the container area A is constant along the height of the container. And we readily set up the discharge from the container:

$$\Delta h = \frac{Q^2}{A_{\mathrm{pipe}}^2 2g}\left(\frac{f_{\mathrm{D}} l}{d} + 1\right)$$

Δh is initially 7.0 m and is reduced with each drop that flows through the pipe. However, each drop that flows from the upper container causes a higher water level in the lower container. Therefore, we establish the following expression for Δh, noting that z is directed downward from the upper water level: $\Delta h = 7 - 2z$. For Q, it follows that:

$$Q = \sqrt{\frac{(7-2z)A_{\mathrm{pipe}}^2 2g}{\left(\frac{f_{\mathrm{D}} l}{d} + 1\right)}}$$

With it, we formulate the continuity condition which, due to the downward coordinate direction z, does not include a negative sign as in Equation 9.20:

$$t = \int_{z_t}^{z_b} \frac{A}{\sqrt{\frac{(7-2z)A_{\mathrm{pipe}}^2 2g}{\left(\frac{f_{\mathrm{D}} l}{d}+1\right)}}}\,\mathrm{d}z$$

We may move the constant part of the expression outside the integral and calculate it separately:

$$\frac{A}{\sqrt{\frac{A_{\mathrm{pipe}}^2 2g}{\left(\frac{f_{\mathrm{D}} l}{d}+1\right)}}} = \frac{100}{\sqrt{\frac{\left(7.85\times 10^{-3}\right)^2 2g}{\left(\frac{0.03\cdot 20}{0.10}+1\right)}}} = 7605.2$$

The integral now looks much more friendly:

$$t = 7605.2 \int_{z_t}^{z_b} \frac{1}{\sqrt{7-2z}}\,\mathrm{d}z = 7605.2 \left[\frac{2}{-2}\sqrt{7-2z}\right]_0^{1.0}$$
$$= 7605.2 \left[-\sqrt{7-2\cdot 1}+\sqrt{7}\right] = 3115.7\,\mathrm{s}$$

The factor 2 results from the integration of the expression with the square root in the denominator, and the division by -2 is due to the term $-2z$.

Solution 2.2

a) With a maximum discharge of $1.0\,\mathrm{m^3/s}$, an impoundment basin of $V = \int Q\,\mathrm{d}t = 1.0\,\mathrm{m^3/s} \cdot 40\,\mathrm{min} \cdot 60\,\mathrm{s/min} = 2400\,\mathrm{m^3}$ is required. With an area of $A = 40 \cdot 20 = 800\,\mathrm{m^2}$, this corresponds to a basin depth of 3.0 m.
The overflow, which must also be designed for the discharge of $1.0\,\mathrm{m^3/s}$, is determined via the Poleni equation (13.25). Solving for h_{WH}, it follows that:

$$h_{\mathrm{WH}} = \left(\frac{3}{2} \cdot \frac{1}{0.60 \cdot 20 \cdot \sqrt{19.62}}\right)^{\frac{2}{3}} = 0.0927\,\mathrm{m}$$

From that we determine the height of the weir:

$$3.0 - h_{\mathrm{WH}} = 2.907\,\mathrm{m}$$

b) The overflow height at minimum discharge is:

$$h_{\mathrm{WH}}(Q_{\min}) = \left(\frac{3 \cdot Q_{\min}}{2 \cdot \mu \cdot b \cdot \sqrt{2g}}\right)^{\frac{2}{3}} = 0.032\,\mathrm{m}$$

This results in a water depth of $h(Q_{\min}) = 2.907\,\mathrm{m} + 0.032\,\mathrm{m} = 2.939\,\mathrm{m}$. Thus, the volume is $V_{\mathrm{basin}} = 20 \cdot 40 \cdot 2.939 = 2351.2\,\mathrm{m^3}$ and:

$$t = \frac{V_{\mathrm{basin}}}{Q_{\min}} = 11\,756\,\mathrm{s} = 196\,\mathrm{min}$$

Solution 2.3

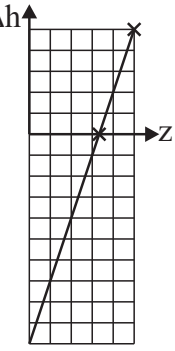
a) The balance is established as follows:

$$h = \frac{1.75 \cdot A_1}{1.5 \cdot A_1} = 1.167\,\mathrm{m}$$

b) From the continuity condition, we obtain:

$$t = -\int_{z_t}^{z_b} \frac{A}{\mu A_O \sqrt{2gz}}\,\mathrm{d}z = -\frac{A}{\mu A_O \sqrt{2g}} \int_{0.75}^{0.25} \frac{1}{\sqrt{z}}\,\mathrm{d}z$$
$$= -694.65 \left[2\sqrt{z}\right]_{0.75}^{0.25} = -694.65 \left[2\sqrt{0.25} - 2\sqrt{0.75}\right] = 508.5\,\mathrm{s}$$

c) This is a really tricky question. Preferably, you draw a coordinate system that shows the height z on the abscissa and the height difference Δh on the ordinate. At $z = 0.25\,\mathrm{m}$, $\Delta h = 0.25\,\mathrm{m}$. The point of intersection of the straight line with the abscissa, where $\Delta h = 0$, lies at $z = 0.167\,\mathrm{m}$. The line intersects the ordinate at $\Delta h = -0.50\,\mathrm{m}$ and the gradient is $3z$. And our equation is hereby established:



$$\Delta h = -0.5 + 3z$$

$$\begin{aligned} t &= -\int_{z_t}^{z_b} \frac{A}{\mu A_O \sqrt{2g\Delta h}}\,\mathrm{d}z = -\frac{A}{\mu A_O \sqrt{2g}} \int_{0.25}^{0.167} \frac{1}{\sqrt{-0.5+3z}}\,\mathrm{d}z \\ &= -694.65 \left[\frac{2}{3}\sqrt{-0.5+3z}\right]_{0.25}^{0.167} \\ &= -694.65 \left[\frac{2}{3}\sqrt{-0.5+3\cdot 0.167} - \frac{2}{3}\sqrt{-0.5+3\cdot 0.25}\right] = 216.9\,\mathrm{s} \end{aligned}$$

By the way, the 2 in the factor $\frac{2}{3}$ results from the division by $\frac{1}{2}$ (from the integration of $z^{-\frac{1}{2}}$) and the 3 from the term $3z$. Thus, the process takes a total of $508.5\,\mathrm{s} + 216.9\,\mathrm{s} = 725.4\,\mathrm{s}$.

Solution 2.4

a) The answer is trivial:

$$t = \frac{A \cdot h}{Q} = 200\,000\,\mathrm{s}$$

b) This is what we obtain from the rearranged Poleni equation:

$$h_{\mathrm{WH}} = \left(\frac{3}{2} \cdot \frac{10}{0.70 \cdot 5 \cdot \sqrt{19.62}}\right)^{\frac{2}{3}} = 0.978\,\mathrm{m}$$

c) Bear in mind that filling takes place in the positive z-direction. This means that the negative sign from Equation 9.19 is not written:

$$t = \int_{z_b}^{z_t} \frac{A}{10 - \frac{2}{3} \cdot \mu \cdot b \cdot \sqrt{2g} \cdot z^{\frac{3}{2}}}\,\mathrm{d}z = \int_0^{0.978} \frac{200\,000}{10 - 10.335 \cdot z^{3/2}}\,\mathrm{d}z$$

This integral may hardly be solved analytically. We use the computer and integrate numerically:

```
1> dz=1e-6;  in order to obtain a reasonable result, the selected
step must be very small.
2> T=0; % for convenience, this is the assumed constant of inte-
gration
3> for z=0:dz:.978 % the loop runs until z=.978 is reached
4> t = 200000/(10 - 10.335 * z^(3/2)) * dz; % the equation for t is
solved for each z as z increases from 0 to 0.978 by dz
5> T=T+t; % T is incremented by t
6> end
7> fname=sprintf('A total of %d s or %d min elapse.',round(T),
round(T/60));
8> disp(fname)
A total of 112305 s or 1872 min elapse.
```

We obtain 3.123×10^5 s together with the already elapsed 2×10^5 s for the time it took to fill the basin to the weir crest.

d) This is easy to calculate. The volume retained "above the weir crest" is:

$$V_{\text{above crest}} = A \cdot h_{\text{WH}} = 200\,000 \cdot 0.978 = 1.956 \times 10^5\,\text{m}^3$$

The following total volume has flowed in during the 1.123×10^5 s:

$$V_{\text{inflow}} = Q \cdot t = 10 \cdot 1.123 \times 10^5 = 1.123 \times 10^6\,\text{m}^3$$

The difference, of course, was spilled into the receiving water:

$$V_{\text{receiving water}} = V_{\text{inflow}} - V_{\text{above crest}} = 9.274 \times 10^5\,\text{m}^3$$

3 Solutions: momentum equation

Solution 3.1

$$v_1 = \frac{Q}{A} = \frac{10}{0.50^2\pi} = 12.73\,\text{m/s};\ \frac{v_1^2}{2g} = 8.26\,\text{m}$$

$$v_2 = \frac{Q}{A} = \frac{5}{0.30^2\pi} = 17.68\,\text{m/s};\ \frac{v_2^2}{2g} = 15.94\,\text{m}$$

$$EGL_1 = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = 38.26\,\text{m}$$

$$EGL_2 = EGL_1 - \Delta e_{1-2} = 38.26 - 2.0 = 36.26\,\text{m}$$

$$\frac{p_2}{\rho g} = EGL_2 - \frac{v_2^2}{2g} = 36.26 - 15.94 = 20.32\,\text{m}$$

$$\begin{aligned} F_R &= p_1 A_1 + \rho Q_1 v_1 - 2\left(p_2 A_2 + \rho Q_2 v_2\right) \\ &= 30 \cdot \rho \cdot g \cdot 0.5^2 \pi + \rho \cdot 10 \cdot 12.73 - 2\left(20.32 \cdot \rho \cdot g \cdot 0.3^2 \pi + \rho \cdot 5 \cdot 17.68\right) \\ &= 6.892 \times 10^4 \,\mathrm{N} \end{aligned}$$

Solution 3.2

a) Consider the Bernoulli equation where $z_1 = z_2$:

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + \frac{p_2}{\rho g}$$

With this equation and the velocities from the continuity condition

$$v_1 = \frac{0.05}{0.2^2 \cdot \pi} = 0.398\,\mathrm{m/s}$$

and

$$v_2 = \frac{0.05}{0.1^2 \cdot \pi} = 1.592\,\mathrm{m/s}$$

we obtain an expression for the pressure height at the outlet:

$$\frac{p_2}{\rho g} = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0.50 + \frac{0.398^2}{2g} - \frac{1.592^2}{2g} = 0.379\,\mathrm{m}$$

Substituting, the force balance becomes:

$$\begin{aligned} F_H &= p_1 A_1 + p_2 A_2 + \rho Q v_1 + \rho Q v_2 \\ &= 0.5 \cdot \rho \cdot g \cdot 0.2^2 \pi + 0.379 \cdot \rho \cdot g \cdot 0.1^2 \pi + \rho \frac{0.05^2}{0.2^2 \pi} + \rho \frac{0.05^2}{0.1^2 \pi} = 832.7\,\mathrm{N} \end{aligned}$$

b) The energy loss is calculated as follows:

$$\Delta e = 0.2\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right) = 0.2 \cdot 0.121 = 0.0242\,\mathrm{m}$$

The energy loss must be subtracted from the pressure height calculated above. Additionally, however, the static pressure of $0.5\,\mathrm{mH_2O}$ from the curvature is in effect.

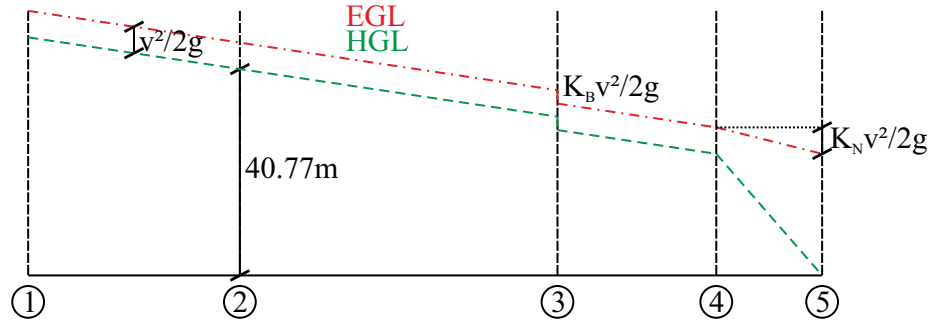
$$\frac{p_2}{\rho g} = 0.379 - 0.0242 + 0.50 = 0.855\,\mathrm{m}$$

The result for the horizontal force is:

$$\begin{aligned} F_H &= p_1 A_1 + p_2 A_2 + \rho Q v_1 + \rho Q v_2 \\ &= 0.5 \cdot \rho \cdot g \cdot 0.2^2 \pi + 0.855 \cdot \rho \cdot g \cdot 0.1^2 \pi + \rho \frac{0.05^2}{0.2^2 \pi} + \rho \frac{0.05^2}{0.1^2 \pi} = 979.3\,\mathrm{N} \end{aligned}$$

Solution 3.3

a)



b) With $4.0\,\text{bar} = 40.77\,\text{mH}_2\text{O}$, it follows that:

$$40.77 + 1.0 + \frac{v^2}{2g} - \frac{v^2}{2g}\left(\frac{f_D l}{d} + K_B\right) - \frac{v^2_{\text{nozzle}}}{2g}\left(K_{\text{nozzle}} + 1\right) = 3.0$$

$$38.77 + \frac{Q^2}{\left(0.05^2\pi\right)^2 2g}\left(1 - \frac{f_D l}{d} - K_B\right) - \frac{Q^2}{\left(0.005^2\pi\right)^2 2g}\left(K_{\text{nozzle}} + 1\right) = 0$$

$$38.77 + \frac{Q^2}{1.21 \times 10^{-3}}\left(1 - \frac{0.03 \cdot 3}{0.10} - 0.8\right) - \frac{Q^2}{1.21 \times 10^{-7}}\left(1.2\right) = 0$$

This yields $Q = 1.977 \times 10^{-3}\,\text{m}^3/\text{s}$.

c) The force with which the nozzle must be held in place corresponds to the momentum flux:

$$F = \rho Q v_{\text{nozzle}} = \frac{1000\left(1.98 \times 10^{-3}\right)^2}{0.005^2\pi} = 49.76\,\text{N}$$

d) To find the attachment force, the pressure conditions upstream from the nozzle must also be known.

$$v = \frac{Q}{A} = 0.252\,\text{m/s};\ \frac{v^2}{2g} = 3.23\,\text{mm}$$

$$v_{\text{nozzle}} = \frac{Q}{A_{\text{nozzle}}} = 25.172\,\text{m/s};\ \frac{v^2_{\text{nozzle}}}{2g} = 32.295\,\text{m}$$

The energy height upstream from the nozzle is:

$$H_{\text{before nozzle}} = z + \frac{v^2_{\text{nozzle}}}{2g} + \Delta e_{\text{nozzle}} = 3.0 + 32.295\left(1 + 0.2\right) = 41.754\,\text{m}$$

The hydraulic grade line (or the piezometric pressure) upstream from the nozzle is:

$$HGL_{\text{before nozzle}} = H_{\text{before nozzle}} - \frac{v^2}{2g} = 41.751\,\text{m};\ \rightarrow \frac{p}{\rho g} = HGL_{\text{before nozzle}} - z = 38.751\,\text{m}$$

This allows the calculation of the force with which the nozzle must be attached to the hose:

$$
\begin{aligned}
F &= \rho Q v + pA - \rho Q v_{\text{nozzle}} \\
&= 1000 \cdot 1.98 \times 10^{-3} \cdot 0.252 + 38.702 \cdot 1000 \cdot 9.81 \cdot 7.85 \times 10^{-3} - 1000 \cdot 1.98 \times 10^{-3} \cdot 25.172 \\
&= 0.499 + 2981.9 - 49.8 = 2932\,\text{N}
\end{aligned}
$$

e) The distance traversed by the water jet depends on the time the jet is in the air.

$$3 = \frac{1}{2} g t^2$$

$$t = 0.782\,\text{s}$$

$$x = v_0 \cdot t = 25.172 \cdot 0.782 = 19.686\,\text{m}$$

Solution 3.4

a) This is our easiest exercise. We calculate the area A_2 with the velocity height and the continuity condition:

$$\frac{v_1^2}{2g} = \frac{4.0^2}{2g} = 0.815\,\text{m} \rightarrow \frac{v_2^2}{2g} = \frac{1}{7}\frac{v_1^2}{2g} = 0.116\,\text{m};\ v_2 = 1.512\,\text{m/s}$$

$$Q = v_1 A_1 = 2.8 \times 10^{-3}\,\text{m}^3/\text{s};\ A_2 = \frac{Q}{v_2} = 1.852 \times 10^{-3}\,\text{m}^2$$

b) When setting up the Bernoulli equation and accounting for energy losses, we obtain:

$$z_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} - \Delta e = z_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\frac{1.0 \times 10^5}{1.0 \times 10^3 \cdot g} + 0.815 - 0.5 \cdot 0.116 = \frac{p_2}{\rho g} + 0.116$$

$$\frac{p_2}{\rho g} = 10.84\,\text{mH}_2\text{O}$$

$$p_2 = 1.063 \times 10^5\,\text{N/m}^2$$

c) Ultimately, this yields the flange force:

$$
\begin{aligned}
F_H &= \rho Q v_1 + p_1 A_1 - \rho Q v_2 - p_2 A_2 \\
&= 1.0 \times 10^3 \cdot 2.8 \times 10^{-3} \cdot 4.0 + 1.0 \times 10^5 \cdot 7 \times 10^{-4} - 1.0 \times 10^3 \cdot 2.8 \times 10^{-3} \cdot 1.512 \\
&- 1.063 \times 10^5 \cdot 1.852 \times 10^{-3} = -119.885\,\text{N}
\end{aligned}
$$

Therefore, F_H acts in the direction of flow.

Solution 3.5

a) Here, the Froude number is required:

$$A = \frac{r^2\pi}{2} = 0.0353\,\mathrm{m}^2$$

$$v = \frac{Q}{A} = 1.132\,\mathrm{m/s}$$

$$\mathrm{Fr} = \frac{v}{\sqrt{g\frac{A}{b_{\mathrm{WS}}}}} = \frac{1.132}{\sqrt{g\frac{0.035}{0.30}}} = 1.054;\ \rightarrow \mathrm{SUPER}$$

b) The centre of gravity of the semicircle is spaced $0.4244 \cdot R$ from the centre of the circle.

$$h'_{\mathrm{CG}} = 0.15 \cdot 0.4244 = 0.0637\,\mathrm{m}$$

$$p_{\mathrm{CG}} = \rho g h'_{\mathrm{CG}} = 624.5\,\mathrm{N/m}^2$$

$$F_P = p_{\mathrm{CG}} A = 624.5 \cdot 0.0353 = 22.05\,\mathrm{N}$$

$$F_M = \rho Q v = 1000 \cdot 0.04 \cdot 1.132 = 45.28\,\mathrm{N}$$

$$F_{MP} = F_M + F_P = 67.33\,\mathrm{N}$$

And because the entering and leaving momentum and pressure forces act on the shaft with an angle of deflection of 90°, the reaction force to be applied is:

$$F_R = \sqrt{F_{MP}^2 + F_{MP}^2} = 95.21\,\mathrm{N}$$

4 Solutions: free surface flows

You may track the individual water level courses via the Bernoulli–Manning discussions, which are not explicitly given here.

Solution 4.1

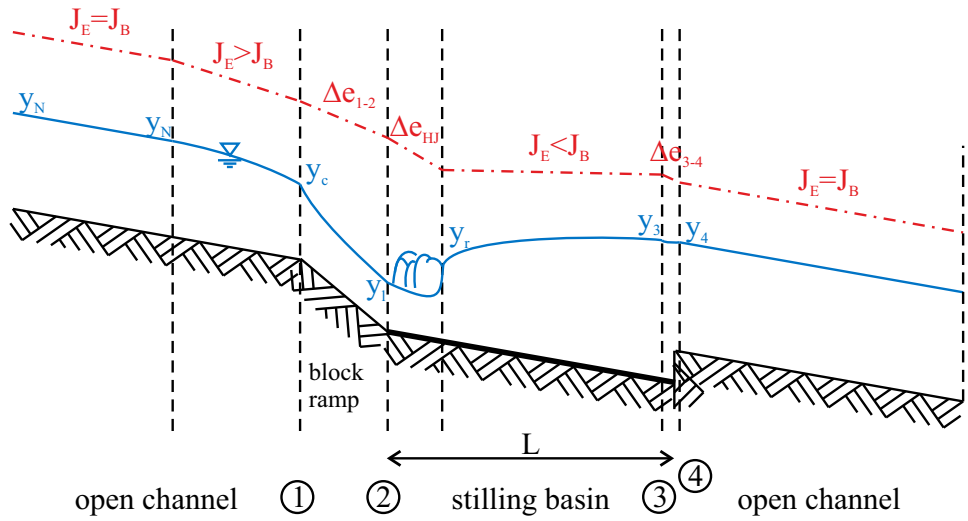
a)

open channel	$y_N = 1.381\,\mathrm{m}$	$v_N = 2.896\,\mathrm{m/s}$	$H_N = 1.809\,\mathrm{m}$	$\mathrm{Fr}_N = 0.787$
	$y_c = 1.177\,\mathrm{m}$	$v_c = 3.398\,\mathrm{m/s}$	$H_{\min} = 1.766\,\mathrm{m}$	$\Delta e_{1-2} = 0.589\,\mathrm{m}$
stilling basin	$y_N = 0.860\,\mathrm{m}$	$v_N = 4.651\,\mathrm{m/s}$	$H_N = 1.963\,\mathrm{m}$	$\mathrm{Fr}_N = 1.601$
	$y_c = 1.177\,\mathrm{m}$	$v_c = 3.398\,\mathrm{m/s}$	$H_{\min} = 1.766\,\mathrm{m}$	$\Delta e_{3-4} = 0.0428\,\mathrm{m}$

b) At point ④, normal water conditions must exist. I leave the Manning–Bernoulli discussion to you (also, see example on page 259). The specific energy at point ③ is $H_3 = H_N + 0.1\frac{v_N^2}{2g} + w_2 = 2.352\,\mathrm{m}$; from the Bernoulli equation, the flow depth that results is $y_3 = 2.181\,\mathrm{m}$. The crucial hint for the water level course is already hidden in the specification. We would normally assume that downstream from the ramp, an increasing flow depth in supercritical flow followed by a hydraulic jump occurs. But we should first find out what's behind that hint; the hydraulic jump is to occur immediately downstream from the ramp without the section where energy decreases in supercritical flow. A specific energy of $H_{\min} + w_1 - \frac{v_c^2}{2g} = 2.677\,\mathrm{m}$ exists downstream from

the ramp[A]. With the Bernoulli equation, we obtain the flow depth in supercritical flow: $y_2 = 0.631\,\text{m}$ at $\text{Fr}_2 = 2.548$. This results in a conjugated flow depth in subcritical flow of $y_{r,\text{HJ}} = 1.980\,\text{m}$ at a specific energy of $H_{r,\text{HJ}} = 2.188\,\text{m}$. Eureka! $H_{r,\text{HJ}} < H_3$. Thus, there must be an energy build-up in the stilling basin. The length of the backwater in the stilling basin is determined by means of the direct step method (`directStepDeltaX.m`) where $y_l = y_{r,\text{HJ}} = 1.980\,\text{m}$ and $y_r = y_3 = 2.181\,\text{m}$ yield $\Delta x = 22.34\,\text{m}$. For the length of the hydraulic jump, we obtain $L_{\text{HJ}} = 4.54\,\text{m}$. The total length of the stilling basin must be $L_{\text{stilling basin}} = L_{\text{HJ}} + \Delta x = 26.88\,\text{m}$[B].

c) The complete course of the water level appears as seen below.



Solution 4.2

a) $\boxed{y_N = 4.026\,\text{m}} \boxed{v_N = 4.140\,\text{m/s}} \boxed{H_N = 4.900\,\text{m}} \boxed{\text{Fr}_N = 0.659}$

b) In the following steps, the characteristic specific energies and flow depths are calculated:

$$q_{\text{cons}} = \frac{Q}{3 \cdot 7.5} = 22.22\,\text{m}^3/(\text{s m})$$

$$H_{\text{min,cons}} = \frac{3}{2}\sqrt[3]{\frac{q_{\text{cons}}^2}{g}} = 5.539\,\text{m}$$

$$y_{c,\text{cons}} = 3.692\,\text{m};\ v_{c,\text{cons}} = 6.019\,\text{m/s};\ \Delta e = 0.5\frac{v_{c,\text{cons}}^2}{2g} = 0.923\,\text{m}$$

$$H_{\text{upstream}} = H_{\text{min,cons}} + \Delta e = 6.462\,\text{m} > H_N \rightarrow \text{energy build-up necessary}$$

The flow depth for $H_{\text{upstream cons}} = 6.462\,\text{m}$ in subcritical flow is obtained with the Bernoulli equation:

[A] Nature adapts to the minimum required energy. H_N is not required until the ramp is reached, but is gradually reduced to H_{min}.

[B] If it were shorter, the energy build-up for the normal water conditions at point ④ would nevertheless be required. The hydraulic jump would "travel into the ramp".

$$y_{\text{upstream cons}} = 6.079\,\text{m};\ v_{\text{upstream cons}} = 2.742\,\text{m/s}$$

As indicated, no further energy loss occurs downstream from the bridge; from the Bernoulli equation, the flow depth in supercritical flow is as follows:

$$H_{\text{downstream cons}} = H_{\text{min}} \rightarrow y_{\text{downstream cons}} = 2.00\,\text{m}$$

The specific energy downstream from the constriction is greater than H_N; therefore, energy is reduced in supercritical flow with increasing flow depth up to $y_{l,\text{HJ}}$ and y_N is in effect immediately downstream from the hydraulic jump.

$$y_{l,\text{HJ}} = \frac{y_N}{2}\left(\sqrt{1+8\text{Fr}_N^2}-1\right) = 2.245\,\text{m}$$

$$H_{l,\text{HJ}} = 5.056\,\text{m};\text{Fr}_{l,\text{HJ}} = 1.58$$

I will skip the energy diagram ;-).

c) We use the direct step method here (`directStepDeltaX.m`). The flow depth on the right corresponds to that upstream from the constriction; the normal depth is to be inserted on the left.

$$\Delta x_{\text{upstream}} = 1150.2\,\text{m}$$

The length of the supercritical section is:

$$\Delta x_{\text{downstream}} = 25.3\,\text{m}$$

Since the Froude number $\text{Fr}_l = 1.58$ upstream from the hydraulic jump is smaller than 1.7, we see an undulating hydraulic jump; here, no statement relating to the length is possible. The total length of the effect may be given as $1175.5\,\text{m} + L_{\text{pier}}$.

Solution 4.3

a) The rearranged Poleni equation reads:

$$h_{\text{WH}} = \left(\frac{3}{2}\cdot\frac{40}{0.60\cdot 8\sqrt{19.62}}\right)^{\frac{2}{3}} = 1.997\,\text{m}$$

$$y_{\text{upstream}} = h_{\text{WH}} + w = 3.497\,\text{m}$$

With this, we calculate the specific energy upstream from the weir:

$$H_{\text{upstream}} = 3.497 + \frac{\left(\frac{40}{8\cdot 3.497}\right)^2}{2g} = 3.601\,\text{m}$$

With Δz added to H_{upstream}, this corresponds to the specific energy $H_{\text{downstream}} = 4.601\,\text{m}$ downstream from the weir because no energy losses are indicated. With $b = 12.0\,\text{m}$ the flow depth in supercritical flow downstream from the weir $y_{\text{downstream}} = 0.366\,\text{m}$ follows from the Bernoulli equation.

b) We know that the additional built-up energy is dissipated again in supercritical flow and that y_N exists immediately downstream from the hydraulic jump.

$$y_{l,\mathrm{HJ}} = \frac{y_{r,\mathrm{HJ}}}{2}\left(\sqrt{1+8\cdot \mathrm{Fr}_r^2}-1\right) = \frac{1.35}{2}\left(\sqrt{1+8\cdot 0.678^2}-1\right) = 0.786\,\mathrm{m}$$

By means of the continuity condition, it follows that $v_l = v_{\mathrm{downstream}} = 9.11\,\mathrm{m/s}$ and $v_r = v_{l,\mathrm{HJ}} = 4.24\,\mathrm{m/s}$; that is, $v_m = 6.67\,\mathrm{m/s}$ and also $R_{\mathrm{hy},m} = 0.526\,\mathrm{m}$. Ultimately, $H_r = H_{l,\mathrm{HJ}} = 1.702\,\mathrm{m}$ and $H_l = H_{\mathrm{downstream}} = 4.601\,\mathrm{m}$. With the Manning equation, we formulate n for normal water conditions

$$n = \frac{\sqrt{J_B}}{Q}\cdot\frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}} = \frac{\sqrt{J_B}}{2.314}$$

and insert the expression for the coefficient into the rearranged direct step method equation; we now have an equation with one unknown, J_B:

$$\begin{aligned} H_r - H_l &= \Delta x\left(J_B - \frac{v_m^2\cdot n^2}{R_{\mathrm{hy},m}^{\frac{4}{3}}}\right) \\ &= \Delta x\left(J_B - \frac{v_m^2\cdot\left(\frac{\sqrt{J_B}}{2.314}\right)^2}{R_{\mathrm{hy},m}^{\frac{4}{3}}}\right) \\ 1.702 - 4.601 &= 20\cdot\left(J_B - \frac{6.67^2\cdot\frac{J_B}{2.314^2}}{0.526^{\frac{4}{3}}}\right) \end{aligned}$$

By means of the Octave equation solver, we obtain: $J_B = 7.79\times 10^{-3}$ and $n = 0.0381\,\mathrm{s/m^{\frac{1}{3}}}$.

Solution 4.4

a) The height difference Δz amounts to $\Delta z = J_B\cdot 5000 + 5\cdot 1.0 = 30\,\mathrm{m}$. In a new flow section of 10 km, a gradient of $J_{B,\mathrm{new}} = 0.3\,\%$ results.

$y_N = 1.331\,\mathrm{m}$	$v_N = 2.253\,\mathrm{m/s}$	$H_N = 1.590\,\mathrm{m}$	$\mathrm{Fr}_N = 0.624$
$y_{N,\mathrm{new}} = 1.086\,\mathrm{m}$	$v_{N,\mathrm{new}} = 1.726\,\mathrm{m/s}$	$H_{N,\mathrm{new}} = 1.238\,\mathrm{m}$	$\mathrm{Fr}_{N,\mathrm{new}} = 0.529$

The flow velocity is lower in the new section so that no bottom steps are required.

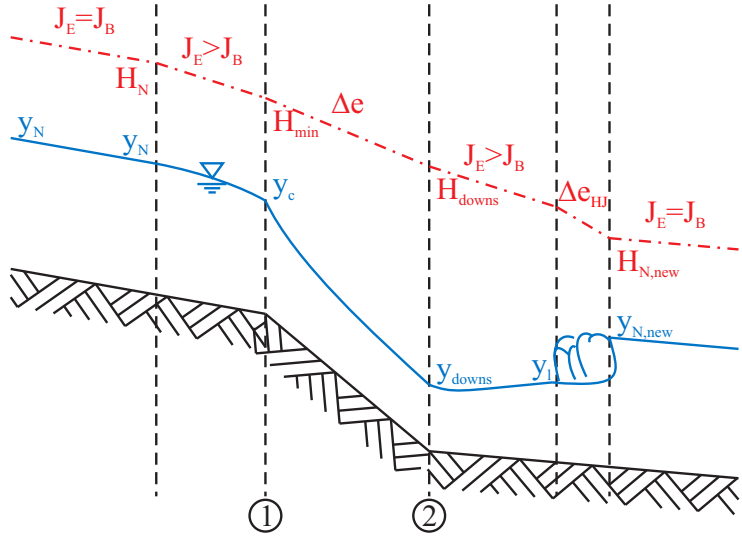
b) At the change in slope, critical conditions prevail with $y_c = 0.972\,\mathrm{m}$ and $H_{\min} = 1.458\,\mathrm{m}$. Downstream from the bottom step, a specific energy of

$$H_{\mathrm{downstream\ step}} = 1.458 + 1.0 - 0.1\frac{v_c^2}{2g} = 2.458 - 0.049 = 2.409\,\mathrm{m}$$

exists. Via the Bernoulli equation, this results in the flow depth $y_{\mathrm{downstream\ step}} = 0.291\,\mathrm{m}$. The supercritical flow depth increases until $y_{l,\mathrm{HJ}} = 0.434\,\mathrm{m}$ is reached.

c) The direct step method yields $\Delta x = 10.25\,\mathrm{m}$. The length of the hydraulic jump is $L_{\mathrm{HJ}} = 1.45\,\mathrm{m}$, which means that a total length of $L_{\mathrm{total}} = 11.7\,\mathrm{m}$ must be paved.

d)



Solution 4.5

a)

section 1	$y_{N,1} = 0.984\,\text{m}$	$v_{N,1} = 2.033\,\text{m/s}$	$H_{N,1} = 1.195\,\text{m}$	$\text{Fr}_{N,1} = 0.654$
section 2	$y_{N,2} = 0.244\,\text{m}$	$v_{N,2} = 2.047\,\text{m/s}$	$H_{N,2} = 0.458\,\text{m}$	$\text{Fr}_{N,2} = 1.323$
section 3	$y_{N,3} = 0.640\,\text{m}$	$v_{N,3} = 0.781\,\text{m/s}$	$H_{N,3} = 0.671\,\text{m}$	$\text{Fr}_{N,3} = 0.311$

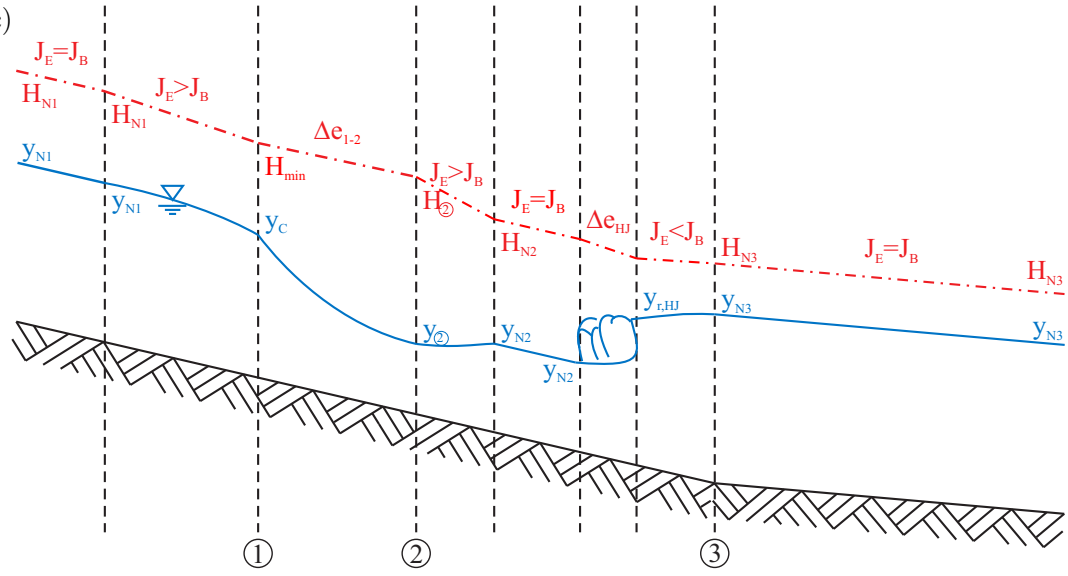
b) The specific energy in section 2 is lower than in section 1. This results in energy reduction. There is no reason for the specific energy to remain at $H_{N,1}$ until the end of the section; nature aims at the lowest energy state. At cross-section ①, $H_{\text{min},1} = 1.11\,\text{m}$ exists at $y_{c,1} = 0.742\,\text{m}$.

The energy loss in the expansion corresponds to the gradient at the bottom in this section. Thus, we obtain $H_2 = 1.11\,\text{m}$ for the specific energy at cross-section ②. Since the critical conditions have already been passed in cross-section ① ($H_{\text{min},1} > H_{\text{min},2}$ and thus crucial), supercritical flow with $y_2 = 0.113\,\text{m}$ is the result. The specific energy in cross-section ② is also too high for the normal conditions of this section; it is continuously reduced to $H_{N,2}$ (or, that is, to $y_{N,2}$ in supercritical flow). This flow depth remains unchanged because it is stated that normal conditions occur in all sections.

However, we see that the specific energy at normal conditions is smaller in section 2 than that in section 3. There must be an energy build-up, which is possible only in subcritical flow. In section 2, there is a hydraulic jump with $y_{N,2}$ on its left. Immediately downstream from the hydraulic jump, the flow depth $y_{r,\text{HJ}} = 0.351\,\text{m}$ is conjugated to $y_{N,2}$.

Downstream from the hydraulic jump – still in section 2 – the energy is built-up. The flow depth increases until it reaches $y_{N,3}$ precisely at the transition to section 3. From there, nothing will change.

c)



Solution 4.6

a)

section 1	$y_{N,1} = 0.836\,\text{m}$	$v_{N,1} = 3.986\,\text{m/s}$	$H_{N,1} = 1.646\,\text{m}$	$\text{Fr}_{N,1} = 1.392$
section 2	$y_{N,2} = 1.647\,\text{m}$	$v_{N,2} = 3.035\,\text{m/s}$	$H_{N,2} = 2.117\,\text{m}$	$\text{Fr}_{N,2} = 0.755$

b) Since $H_{N1} < H_{N2}$, H_{N2} is found immediately downstream from the constriction. Together with $\Delta e_{\text{cons}} = 0.0469\,\text{m}$ and $\Delta z = 5.0 \cdot J_{B2} = 0.015\,\text{m}$, this results in:

$$H_A = H_{\text{upstream cons}} = H_{N2} + \Delta e_{\text{cons}} - \Delta z = 2.149\,\text{m}$$

With this specific energy, we may calculate the flow depth upstream from the constriction by means of the Bernoulli equation: $y_{\text{upstream cons}} = y_A = 2.009\,\text{m}$.
Initially, normal conditions occur in section 1 until a transition to subcritical flow takes place ($y_{r,\text{HJ}} = 1.280\,\text{m}$). From there, energy is built-up until, immediately upstream from the constriction, $y_{\text{upstream cons}} = 2.009\,\text{m}$ (that is $H_{\text{upstream cons}} = 2.149\,\text{m}$) is reached. This is just enough for achieving normal conditions with the energy losses in the constriction immediately at the beginning of section 2.

c) I'm sure you can manage the energy diagram yourself.

5 Solutions: pipe flow

Solution 5.1

a) Initially, we establish the energy equation:

$$H_A - \frac{Q^2}{A_1^2 2g}\left(\frac{f_{\text{D1}} L_1}{d_1}\right) - \frac{Q^2}{A_2^2 2g}\left(\frac{f_{\text{D2}} L_2}{d_2} + 1\right) - H_T = H_B$$

With the specified values, it follows that:

$$H_A - \frac{Q^2}{0.785^2 2g}\left(\frac{0.025\cdot 2000}{1.0}\right) - \frac{Q^2}{1.767^2 2g}\left(\frac{0.020\cdot 1000}{1.5}+1\right) - H_T = H_B$$

With the power equation for the turbine, we may write:

$$H_T = \frac{P_T}{\eta\rho g Q} = \frac{P_T}{0.80\cdot 9.81\cdot 1000\cdot Q} = 750 - 4.131\cdot Q^2 - 0.234\cdot Q^2$$

Consequently:

$$P_T = \left(750 - 4.365\cdot Q^2\right)\cdot 7848\cdot Q = 5.886\times 10^6\cdot Q - 3.426\times 10^4\cdot Q^3$$

The maximum value of P_T is found by differentiation:

$$\frac{\mathrm{d}P_T}{\mathrm{d}Q} = 5.886\times 10^6 - 3\cdot 3.426\times 10^4\cdot Q^2 \stackrel{!}{=} 0$$

Finally, we obtain $P_{T,\max}$ when $Q = 7.568\,\mathrm{m^3/s}$.

b) The above established power equation for the turbine yields H_T:

$$H_T = 750 - 4.365\cdot Q^2 = 750 - 4.365\cdot 7.568^2 = 500.0\,\mathrm{m}$$

c) Finally, we calculate the turbine power:

$$P_T = \eta\rho g Q H_T = 0.80\cdot\rho\cdot g\cdot 7.568\cdot 500 = 29.7\,\mathrm{MW}$$

Solution 5.2

a) For determination of the pump power, we set up the energy equation and determine the pipe friction coefficient for the required discharge.

$$v = \frac{Q}{A} = 0.396\,\mathrm{m/s}$$

With `pc('circle',1.5,1.5,0.7,3e-3,10)`, we find that $f_\mathrm{D} = 0.024$.

$$H_A - \frac{v^2}{2g}\left(\frac{f_\mathrm{D} L}{d} + K_I + 2K_{45^\circ} + 1\right) + H_P = H_B$$

$$H_P = 10 + 0.656 = 10.656\,\mathrm{m};\; P_P = \frac{1}{\eta_P}\rho g Q H_P = 91.47\,\mathrm{kW}$$

This results in the pump costs per year:

$$C_P = P_P\,(10\cdot 365)\cdot 0.15 = 50080\,€/\mathrm{a}$$

b) In this case, too, we cannot bypass the energy equation.

$$H_P = 10 + \frac{16 \cdot Q^2}{d^4 \pi^2 2g}\left(\frac{f_D L}{d} + 1.1 + 1\right) = 10 + \frac{0.0405}{d^4}\left(\frac{120}{d} + 2.1\right)$$

$$P_P = \frac{1}{\eta_P}\rho g Q H_P = 8583.8 \cdot H_P$$

P_P is calculated in watts; however, the costs are indicated in €/kW/h; therefore, we must divide by 1000:

$$C_{P,30} = 0.15\frac{1}{1000}(10 \cdot 365 \cdot 30) \cdot 8583.8 \cdot H_P = 140988 \cdot H_P$$

$$C_{\text{invest}} = L \cdot 7000 \cdot d = 3.5 \times 10^7 \cdot d$$

$$C_{\text{total}} = C_{P,30} + C_{\text{invest}} = 140988\left(10 + \frac{0.0405}{d^4}\left(\frac{120}{d} + 2.1\right)\right) + 3.5 \times 10^7 \cdot d$$

$$1.410 \times 10^6 + \frac{6.852 \times 10^5}{d^5} + \frac{1.199 \times 10^4}{d^4} + 3.5 \times 10^7 \cdot d$$

The total cost must be differentiated with respect to the diameter and set to zero:

$$\frac{\mathrm{d}C_{\text{total}}}{\mathrm{d}d} = 3.5 \times 10^7 - 5 \cdot \frac{6.852 \times 10^5}{d^6} - 4 \cdot \frac{1.199 \times 10^4}{d^5} \stackrel{!}{=} 0$$

With `function C=f(d)` and the above equation, we obtain with `[d,fval,info]=fsolve(@f,.4)` an optimum diameter of:

$$d_{\text{opt}} = 0.680\,\text{m}$$

Solution 5.3

a) First, let's set up the energy equation:

$$2000 - \frac{Q^2}{A^2 2g}\left(\frac{f_D L}{d} + \sum_{i=1}^{n} K_i + K_O\right) - H_T = 1500$$

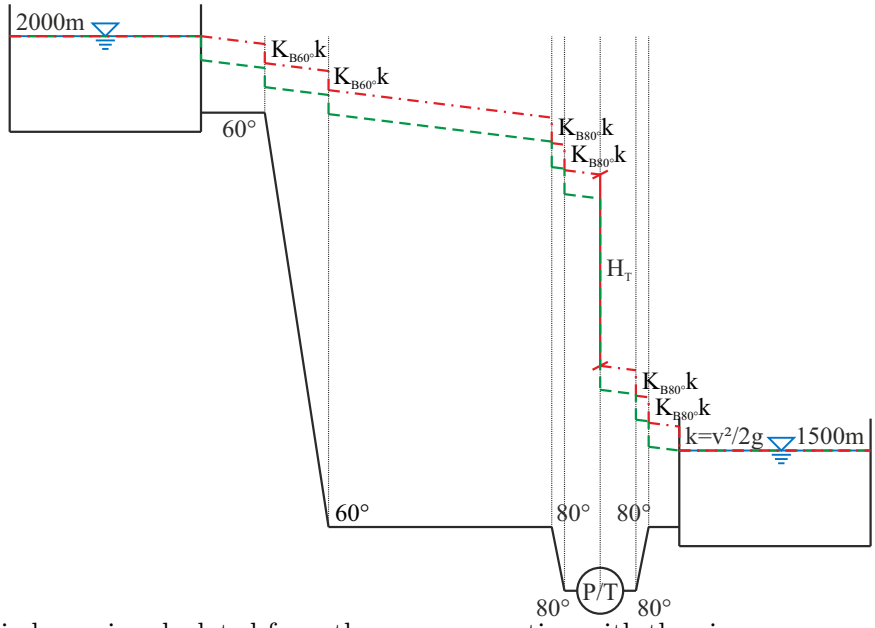
Furthermore:

$$2000 - \frac{Q^2}{3.142^2 2g}\left(\frac{f_D \cdot 5000}{2.0} + 2 \cdot K_{B,60^\circ} + 4 \cdot K_{B,80^\circ} + 1\right) - H_T = 1500$$

$$500 - \frac{Q^2}{3.142^2 2g}(2500 f_D + 4.8) - \frac{92.9 \times 10^6}{0.9 \cdot \rho \cdot g \cdot Q} = 0$$

With $f_{\text{D,step 0}} = 0.010$, we obtain a discharge of $Q_{\text{step 0}} = 27.32\,\text{m}^3/\text{s}$. From the Prandtl-Colebrook algorithm, $f_{\text{D,step 1}} = 0.0108$ results from $Q_{\text{step 0}}$. When inserted into the energy equation, Q becomes $Q_{\text{step 1}} = 29.32\,\text{m}^3/\text{s}$, with which the friction coefficient is again determined via the Prandtl-Colebrook equations. We obtain $f_{\text{D,step 2}} = 0.0108$ and may stop at this point. We calculated with the correct f_D.

b)



c) The discharge is calculated from the energy equation with the given pump power:

$$\frac{\eta \cdot 50 \times 10^6}{\rho \cdot g \cdot Q} = \frac{0.90 \cdot 50 \times 10^6}{1000 \cdot 9.81 \cdot Q} = 500 + \frac{Q^2}{3.142^2 2g}(2500 \cdot 0.0108 + 4.8)$$

We obtain $Q = 8.94\,\mathrm{m^3/s}$.

d) Here, the geodetic height must be taken into consideration since the total pressure (the sum of atmospheric pressure and water pressure) must not drop below the vapour pressure.

$$z_{\max} + \frac{p_{\min}}{\rho g} + \frac{v_{\max}^2}{2g} = 1500$$

With $\frac{v_{\max}^2}{2g} = 31.855\,\mathrm{m}$ and $\frac{p_{\min}}{\rho g} = -7.5\,\mathrm{mH_2O}$, we find that $z_{\max} = 1475.6\,\mathrm{m.a.s.l.}$

Solution 5.4

a) The solution starts again with the energy equation:

$$570 - \frac{Q^2}{A^2 2g}\left(\frac{f_D L}{d} + 1.5\right) = 500$$

With $f_{D,\text{step }0} = 0.020$, $Q_{\text{step }0} = 1.947\,\mathrm{m^3/s}$; subsequently, $f_{D,\text{step }1} = 0.0161$ and $Q_{\text{step }1} = 2.169\,\mathrm{m^3/s}$; $f_{D,\text{step }2} = 0.0160$ and $Q_{\text{step }2} = 2.172\,\mathrm{m^3/s}$. This confirms and ends the process because $f_{D,\text{step }3} = 0.0160$.

b) We set up the following system of equations:

$$560 + H_{P1} - \frac{Q_1^2}{A^2 2g}\left(\frac{f_D L_1}{d} - 1\right) = \mathrm{HGL}_{\mathrm{junct}}$$
$$560 + H_{P2} - \frac{Q_2^2}{A^2 2g}\left(\frac{f_D L_2}{d} - 1\right) = \mathrm{HGL}_{\mathrm{junct}}$$
$$\mathrm{HGL}_{\mathrm{junct}} - \frac{(Q_1 + Q_2)^2}{A^2 2g}\left(\frac{f_D L_3}{d}\right) = 570$$

Substituting values, and with $Q_1 + Q_2 = 2.172\,\mathrm{m}^3/\mathrm{s}$, the result is the following:

$$560 + \frac{P_{P1} \cdot 0.7}{\rho g Q_1} - \frac{Q_1^2}{1.539^2 2g}\left(\frac{0.0160 \cdot 1100}{1.40} + 1\right) = \mathrm{HGL}_{\mathrm{junct}}$$
$$560 + \frac{P_{P2} \cdot 0.7}{\rho g Q_2} - \frac{Q_2^2}{1.539^2 2g}\left(\frac{0.0160 \cdot 100}{1.40} + 1\right) = \mathrm{HGL}_{\mathrm{junct}}$$
$$\mathrm{HGL}_{\mathrm{junct}} - \frac{2.172^2}{1.539^2 2g}\left(\frac{0.0160 \cdot 3000}{1.40}\right) = 570$$

With the third equation, we may quite comfortably calculate the piezometric pressure height at the junction: $\mathrm{HGL}_{\mathrm{junct}} = 573.48\,\mathrm{m.a.s.l.}$ With $P_{P1} = P_{P2} = P_P$, we solve the first two equations for P_P, as follows, and then substitute $Q_2 = 2.172 - Q_1$ in the second equation to obtain the system as seen here:

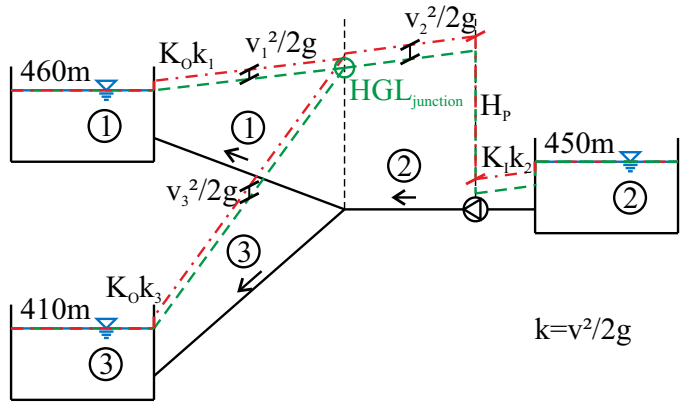
$$P_P = \frac{\rho g Q_1}{0.7}\left(13.48 + \frac{Q_1^2}{1.539^2 2g}\left(\frac{0.0160 \cdot 1100}{1.40} - 1\right)\right)$$
$$P_P = \frac{\rho g Q_2}{0.7}\left(13.48 + \frac{Q_2^2}{1.539^2 2g}\left(\frac{0.0160 \cdot 100}{1.40} - 1\right)\right)$$
$$= \frac{\rho g\,(2.172 - Q_1)}{0.7}\left(13.48 + \frac{(2.172 - Q_1)^2}{1.539^2 2g}\left(\frac{0.0160 \cdot 100}{1.40} - 1\right)\right)$$

By equating the expressions for P_P[C], we ultimately determine that $Q_1 = 1.0748\,\mathrm{m}^3/\mathrm{s}$ and $Q_2 = 1.0972\,\mathrm{m}^3/\mathrm{s}$. Likewise, we find that $P_P = 208.12\,\mathrm{kW}$, $H_{P1} = 13.817\,\mathrm{m}$ and $H_{P2} = 13.536\,\mathrm{m}$.

[C] Preferably, use the Octave solver again and equate the two expressions for P_P since the pump powers are identical.

Solution 5.5

a)



b) This is the system of equations with the boundary condition being that the hydraulic gradient lines intersect at the junction:

$$450 + H_P - \frac{(Q_1+Q_3)^2}{R_2^4\pi^2 2g}\left(\frac{f_{D2}l_2}{d_2} + K_I + 1\right) = \mathrm{HGL_{junct}}$$

$$P_P = \frac{1}{\eta_P}\rho g Q H_P$$

$$H_P = \frac{5.0\times10^5\cdot 0.85}{1.0\times10^3\cdot 9.81\cdot Q_2} = \frac{43.32}{Q_2}$$

$$450 + \frac{43.32}{Q_2} - 3.430\cdot(Q_1+Q_3)^2 = \mathrm{HGL_{junct}}$$

$$\mathrm{HGL_{junct}} - \frac{Q_1^2}{R_1^4\pi^2 2g}\left(\frac{f_{D1}l_1}{d_1}\right) = 460; \rightarrow Q_1 = \sqrt{(\mathrm{HGL_{junct}} - 460)\cdot 0.224}$$

$$\mathrm{HGL_{junct}} - \frac{Q_3^2}{R_3^4\pi^2 2g}\left(\frac{f_{D3}l_3}{d_3}\right) = 410 \rightarrow Q_3 = \sqrt{(\mathrm{HGL_{junct}} - 410)\cdot 7.35\times10^{-4}}$$

$$450 + \frac{43.32}{\sqrt{(\mathrm{HGL_{junct}}-460)\cdot 0.224} + \sqrt{(\mathrm{HGL_{junct}}-410)\cdot 7.35\times10^{-4}}} - 3.430\cdot$$
$$\cdot\left(\sqrt{(\mathrm{HGL_{junct}}-460)\cdot 0.224} + \sqrt{(\mathrm{HGL_{junct}}-410)\cdot 7.35\times10^{-4}}\right)^2 = \mathrm{HGL_{junct}} \tag{17.2}$$

When entering this equation into Octave, one thankfully obtains the following for the piezometric pressure height at the junction:

$$\mathrm{HGL_{junct}} = 468.55\,\mathrm{m}$$

This results in

$$Q_1 = \sqrt{(\mathrm{HGL_{junct}} - 460)\cdot 0.224} = 1.384\,\mathrm{m^3/s}$$

and:

$$Q_2 = \sqrt{(\mathrm{HGL_{junct}} - 410)\cdot 7.35\times10^{-4}} = 0.207\,\mathrm{m^3/s}$$

c) A rule of proportion is required.

$$\frac{1.384}{0.207} \cdot 10000 = 6.686 \times 10^4\,\mathrm{m}^3$$

d) Now the question remains: For how long must the pump operate?

$$t = \frac{V_3}{Q_3} = \frac{10000}{0.207} = 4.83 \times 10^4\,\mathrm{s} = 13.42\,\mathrm{h}$$

$$C_{\mathrm{pump}} = 500 \cdot 13.42 \cdot 0.20 = 1342.0\,€/\mathrm{d}$$

6 Solutions: cross-cutting issues

Solution 6.1

a) The energy equation is required:

$$3.0 = \frac{\left(0.9 \times 10^{-3}\right)^2}{0.02^4 \cdot \pi^2 \cdot 2g}\left(\frac{0.02 \cdot 100}{0.04} + 1 + K_V\right)$$

This results in $K_V = 63.75$.

b) We now use the continuity condition in the form $Q\,\mathrm{d}t = -A\,\mathrm{d}z$:

$$Q(z) = \sqrt{\frac{\Delta h}{1.646 \times 10^6}} = 7.794 \times 10^{-4} \cdot \sqrt{1.85 + z}$$

$$t = -\int_{1.15}^{0} \frac{A_A}{Q_{\mathrm{outflow}} - Q_{\mathrm{inflow}}}\,\mathrm{d}z = -\int_{1.15}^{0} \frac{1.35}{7.794 \times 10^{-4} \cdot \sqrt{1.85 + z} - 0.9 \times 10^{-3}}\,\mathrm{d}z = 5428\,\mathrm{s}$$

We may solve the integral numerically by means of Octave:

```
1> dz=1e-3; % the equation is solved millimetre by millimetre
2> T=0; % this is quasi the integration constant
3> for z=0:dz:1.15 % the loop runs until z=1.15 is reached
4> t = 1.35/(7.794e-4*sqrt(1.85+z)-.9e-3)*dz; % the equation is
solved for each z
5> T=T+t; % the time increment is added to T
6> end
7> fname=sprintf('A total of %d s or %d min elapse.',round(T),
round(T/60));
8> disp(fname)
A total of 5428 s or 90 min elapse.
```

Solution 6.2

a) The energy supplied by the pump must be sufficient to overcome the static head and the hydraulic losses:

$$H_P = 20 + \frac{Q^2}{A^2 2g}\left(\frac{f_D L}{d} + K_B + 1\right)$$

We proceed with the power formula for pumps:

$$\frac{7500 \cdot 0.8}{\rho g Q} = 20 + \frac{Q^2}{0.1257^2 \cdot 2g}\left(\frac{f_D \cdot 8000}{0.40} + 0.4 + 1\right)$$

For $f_{D,\text{step }0} = 0.020$, it follows that $Q_{\text{step }0} = 0.0290\,\text{m}^3/\text{s}$; then $f_{D,\text{step }1} = 0.0272$ and $Q_{\text{step }1} = 0.0285\,\text{m}^3/\text{s}$; $f_{D,\text{step }2} = 0.0272$, which confirms and ends the process, and the discharge is the same as with $f_{D,\text{step }1}$.

b) The function therefore generally reads:

$$\frac{7500 \cdot 0.8}{\rho g\left(\pi \cdot k_f \frac{h_1^2 - h_0^2}{\ln\left(\frac{r_1}{r_0}\right)}\right)} = (20 + h_1 - h_0) + \frac{\left(\pi \cdot k_f \frac{h_1^2 - h_0^2}{\ln\left(\frac{r_1}{r_0}\right)}\right)^2}{0.1257^2 \cdot 2g}\left(\frac{0.0272 \cdot 8000}{0.40} + 0.4 + 1\right)$$

Substituting values:

$$\frac{7500 \cdot 0.8}{\rho g\left(\pi \cdot 1.0 \times 10^{-4} \frac{40^2 - h_0^2}{\ln\left(\frac{3000}{0.2}\right)}\right)} = (60 - h_0) + \frac{\left(\pi \cdot 1.0 \times 10^{-4} \frac{40^2 - h_0^2}{\ln\left(\frac{3000}{0.2}\right)}\right)^2}{0.1257^2 \cdot 2g}\left(\frac{0.0272 \cdot 8000}{0.40} + 1.4\right)$$

And finally one obtains, thanks to Octave, $h_0 = 31.119\,\text{m}$.

c) We have only to insert h_0:

$$Q = \pi \cdot k_f \frac{h_1^2 - h_0^2}{\ln\left(\frac{r_1}{r_0}\right)} = 0.0206\,\text{m}^3/\text{s}$$

d) This is the difference between h_1 and h_0:

$$\Delta h = h_1 - h_0 = 8.881\,\text{m}$$

Solution 6.3

a) With

$$60 = \frac{Q^2}{A^2 2g}\left(\frac{f_D L}{d_{\text{pipe}}} + 1\right)$$

$Q = 0.376\,\text{m}^3/\text{s}$ and $v = 1.915\,\text{m/s}$; hence, $\frac{v^2}{2g} = 0.187\,\text{m}$ is the result.

$$p_{\text{piezo@air vessel}} = 500 + \frac{60 - 0.187}{2} = 529.91\,\text{m.a.s.l. at } z_{\text{pipe@air vessel}} = 520\,\text{m.a.s.l.}$$

This is to determine the total pressure in the pipe at the air vessel:

$$p_{\text{total@air vessel}} = \rho \cdot g \cdot 9.91 + p_0 = 1.9852 \times 10^5\,\text{N/m}^2$$

b) The air volume in the air vessel is:

$$V_{\text{air vessel}} = \frac{4}{3}R^3\pi + h \cdot R^2\pi + 0.1^2 \cdot \pi \cdot 0.5 = 1.717\,\text{m}^3$$

$$V_1 = \frac{p_0 V_0}{p_1} = \frac{101\,300 \cdot 1.717}{1.9852 \times 10^5} = 0.876\,\text{m}^3$$

In the upper hemisphere, the volume is $\frac{2}{3}R^3 \cdot \pi = 0.262\,\text{m}^3$; it is $0.876 - 0.262 = 0.614\,\text{m}^3$ in the remaining part. From the contact point of the upper hemisphere, $h' = \frac{0.614}{R^2\pi} = 0.782\,\text{m}$. The water level in the air vessel rises up to $520 + 0.5 + 0.5 + 1.5 - 0.782 = 521.72\,\text{m.a.s.l.}$

c) The water level rises. Due to the lower discharge, the losses in the pipe section to the valve are lower; therefore, the pressure height is greater and the air volume is further compressed.

Solution 6.4

a)

$y_N = 0.804\,\text{m}$	$v_N = 5.328\,\text{m/s}$	$H_N = 2.252\,\text{m}$
$\text{Fr}_N = 1.897$	$y_c = 1.233\,\text{m}$	$H_{\min} = 1.849\,\text{m}$

We are in supercritical flow and $H_N - K\frac{v_N^2}{2g} > H_{\min}$. Thus, there will be no backwater and the energy loss occurs at the trash rack. With $H_{\text{downs}} = H_N - K\frac{v_N^2}{2g} = 1.963\,\text{m}$, $y_{\text{downs}} = 0.972\,\text{m}$ follows (also supercritical flow).

b) Here again, the momentum equation is the means of choice. The control volume extends from upstream to downstream from the rack.

$$F + \rho \cdot \frac{Q^2}{b \cdot y_N} + \frac{1}{2} \cdot \rho \cdot g \cdot y_N^2 \cdot b = \rho \cdot \frac{Q^2}{b \cdot y_{\text{downs}}} + \frac{1}{2} \cdot \rho g \cdot y_{\text{downs}}^2 \cdot b$$

$$\rightarrow F = \rho \cdot \frac{30^2}{7 \cdot 0.972} + \frac{1}{2} \cdot \rho \cdot g \cdot 0.972^2 \cdot 7 - \rho \cdot \frac{30^2}{7 \cdot 0.804} - \frac{1}{2} \cdot \rho \cdot g \cdot 0.804^2 \cdot 7 = -1.740 \times 10^4\,\text{N}$$

7 Solutions: unsteady free surface flows

Solution 7.1

a) Velocity is distance per unit time.

$$c_{\text{abs}} = \frac{x}{t} = \frac{2000}{589} = 3.396\,\text{m/s}$$

b) We calculate the relative propagation velocity with surge height $h = 0.45\,\text{m}$.

$$c = \sqrt{\frac{g \cdot 7.5 \cdot 2.1}{7.5}} \cdot \sqrt{1 + \frac{3}{2} \cdot \frac{0.45 \cdot 7.5}{7.5 \cdot 2.1} + \frac{1}{2}\left(\frac{0.45 \cdot 7.5}{7.5 \cdot 2.1}\right)^2} = 5.26\,\text{m/s}$$

Only now we see that the absolute propagation velocity has a negative sign. This means that:

$$c_{\text{abs}} = \frac{x}{t} = \frac{-2000}{589} = -3.396\,\text{m/s}$$

But we have to go back to the calculation of the original flow velocity v_0

$$v_0 = c_{\text{abs}} + c = -3.40 + 5.26 = 1.86\,\text{m/s}$$

and to the discharge Q_0:

$$Q_0 = v_0 A_0 = 29.30\,\text{m}^3/\text{s}$$

c) The reduction in discharge results from the continuity condition:

$$\Delta Q = c_{\text{abs}} \cdot h \cdot b = -3.40 \cdot 0.45 \cdot 7.50 = -11.475\,\text{m}^3/\text{s}$$

d) It is an upsurge.

Solution 7.2

a) With the routine `surge.m`, the result reads[D]:

$$c = 8.05\,\text{m/s}$$

$$c_{\text{abs}} = -7.03\,\text{m/s}$$

b) We determine the height of the surge:

$$h_{\text{upsurge}} = 0.395\,\text{m}$$

Thus, the freeboard is maintained because $1.5 - 0.395 > 1.0$.

c) Time corresponds to distance divided by velocity.

$$t = \frac{x}{c_{\text{abs}}} = 1138\,\text{s} = 18.97\,\text{min}$$

Solution 7.3

a) And it is the continuity condition which saves us yet again:

[D] Please note that you may display the associated propagation velocities after the `while` loop via `disp(c)`; the routine displays the result for h only.

$$v_0 = \frac{Q_0}{A_0} = \frac{5.0 \times 10^{-3}}{0.20 \cdot 0.05} = 0.50\,\mathrm{m/s}$$

b) A clever technique!

$$h = 0.04\,\mathrm{m}$$

c) We once more set up the equation for the propagation velocity of shallow water waves, which are affected by the surge height h.

$$c = \sqrt{\frac{g \cdot 0.20 \cdot 0.05}{0.20}} \sqrt{1 + \frac{3}{2} \cdot \frac{0.04 \cdot 0.20}{0.20 \cdot 0.05} + \frac{1}{2} \left(\frac{0.04 \cdot 0.20}{0.20 \cdot 0.05} \right)^2} = 1.11\,\mathrm{m/s}$$

d) The propagation of the surge wave takes place opposite ($-c$) to the direction of flow $+v_0$:

$$c_{\mathrm{abs}} = v_0 - c = 0.50 - 1.11 = -0.61\,\mathrm{m/s}$$

e) time = distance / velocity:

$$t = \frac{x}{c_{\mathrm{abs}}} = \frac{-1.38}{-0.61} = 2.26\,\mathrm{s}$$

The result is 2.26 s after the closing of the open channel.

f) The continuity condition is necessary:

$$\Delta Q = 0.04 \cdot (-0.61) \cdot 0.20 = -4.88\,\mathrm{L/s}$$

The discharge reduction amounted to −97.6 %; i. e. the sluice gate was (almost) completely closed.

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